

# *Fundamentals of (Static and Dynamic) Software Verification*

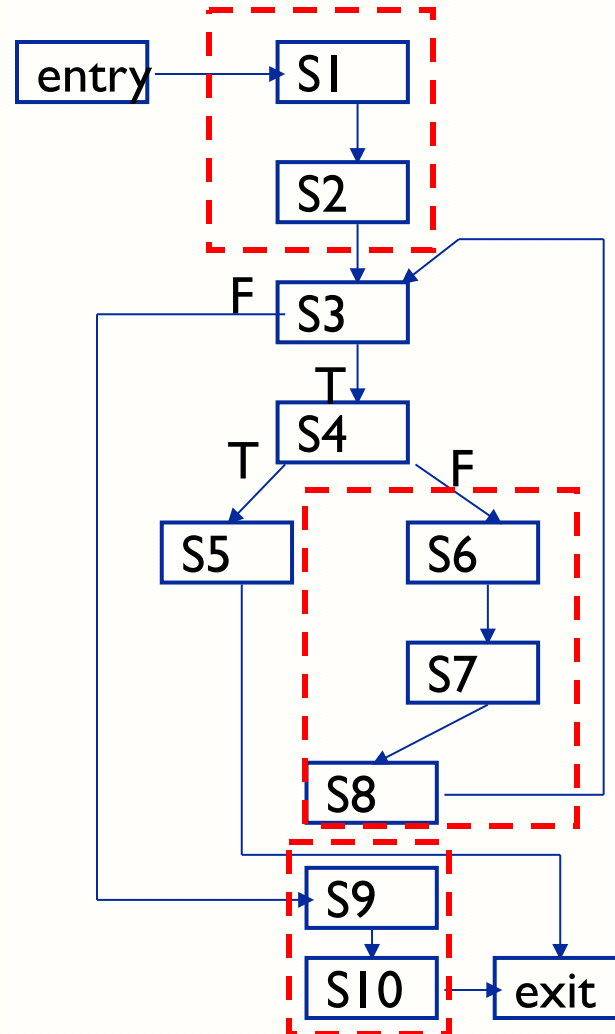
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*Control-flow Analysis*  
*Data-flow Analysis*  
*Static VS Dynamic Analysis*  
*Software Testing*

# Control Flow Graph

## Procedure AVG

```
S1  count = 0
S2  fread(fp_ptr, n)
S3  while (not EOF) do
S4    if (n < 0)
S5      return (error)
    else
S6      nums[count] = n
S7      count ++
    endif
S8    fread(fp_ptr, n)
  endwhile
S9  avg = mean(nums, count)
S10 return(avg)
```

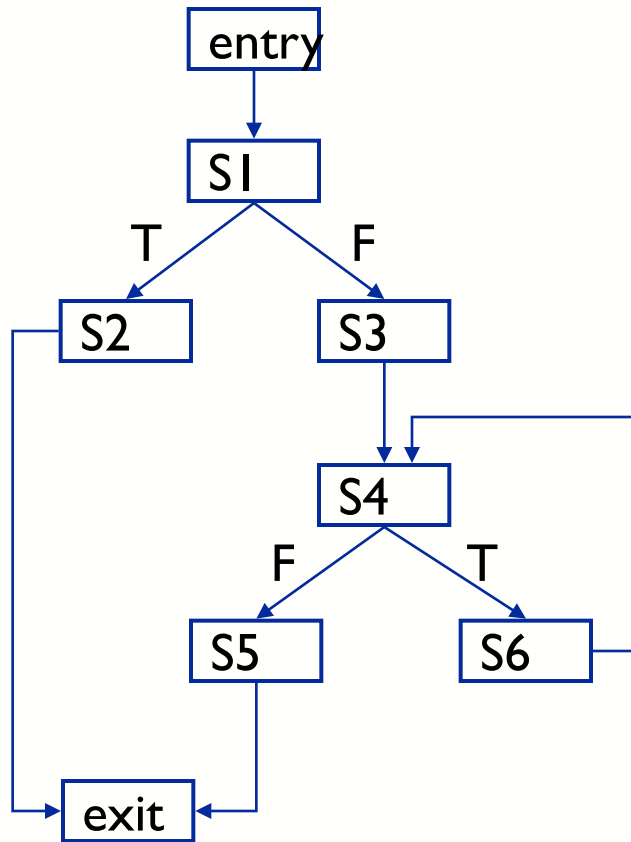


# Dominance/Postdominance

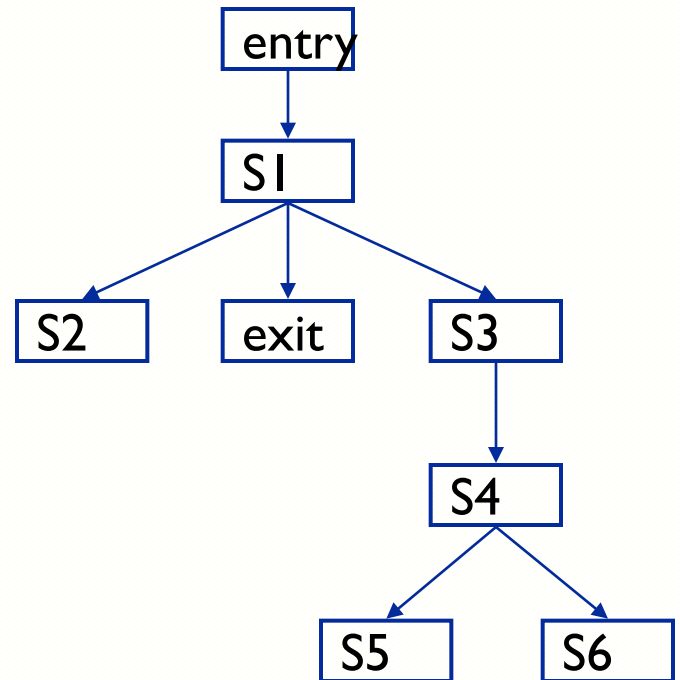
- A node  $n$  **dominates** a node  $m$  ( $n \text{ dom } m$ ) if every path from the entry to  $m$  includes  $n$
- A node  $n$  **postdominates** a node  $m$  ( $n \text{ pdom } m$ ) if every path from the  $m$  to the exit includes  $n$

# *Dominance*

**CFG**



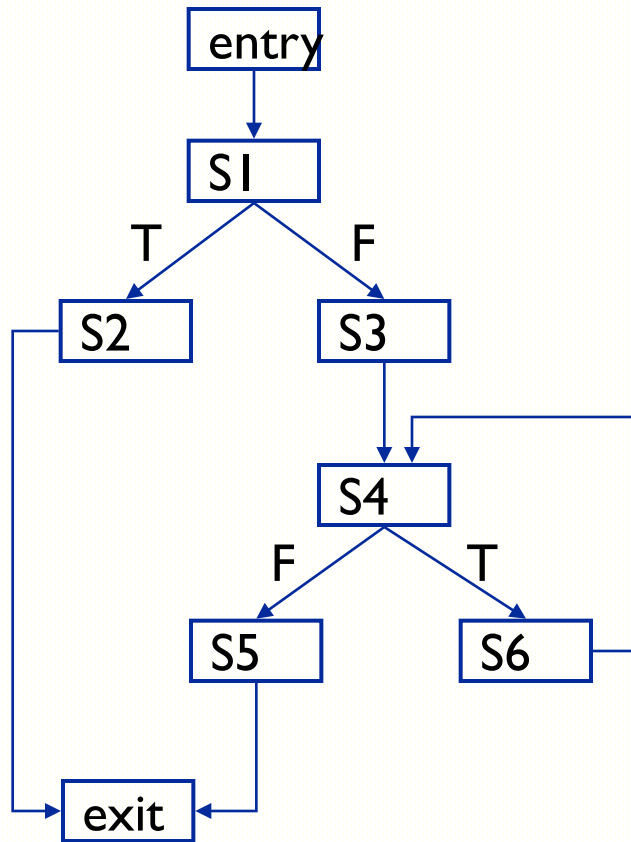
**Dominance Tree**



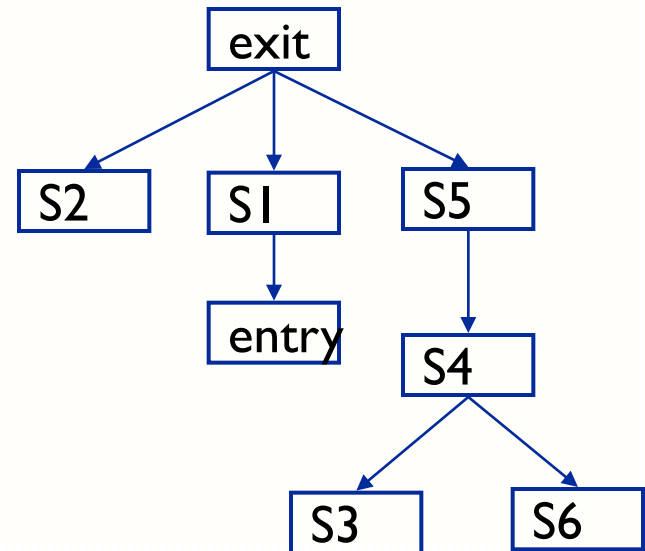


# Postdominance

## CFG



## Postdominance Tree



# Basic Data-Flow concepts

- **Definition and Use**

Consider statement  $X = Y + Z$

- Definitions?
- Uses?

- **Kill and Reach**

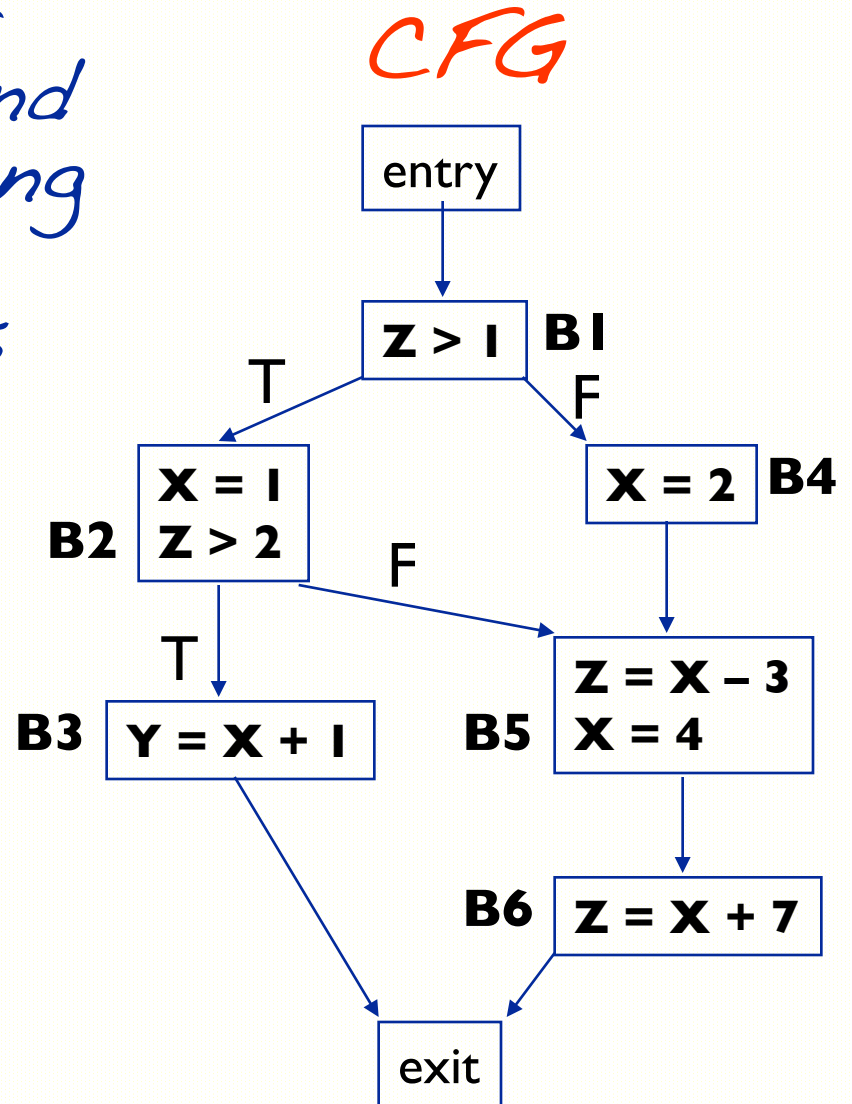
- A definition  $d$  of a variable  $x$  is **killed** at a statement  $s$  iff  $s$  redefines  $x$  and the last assignment to  $x$  was  $d$
- A definition  $d$  of  $x$  **reaches**  $s$  if there is at least a path from  $d$  to  $s$  along which  $x$  is not killed (**def-clear path**)

# Dependence Analysis

- Important for
  - Optimization  
(e.g., instruction scheduling)
  - Software engineering
    - Program understanding
    - Reverse engineering
    - Debugging
- Two main kinds of dependences
  - Data related
  - Control related

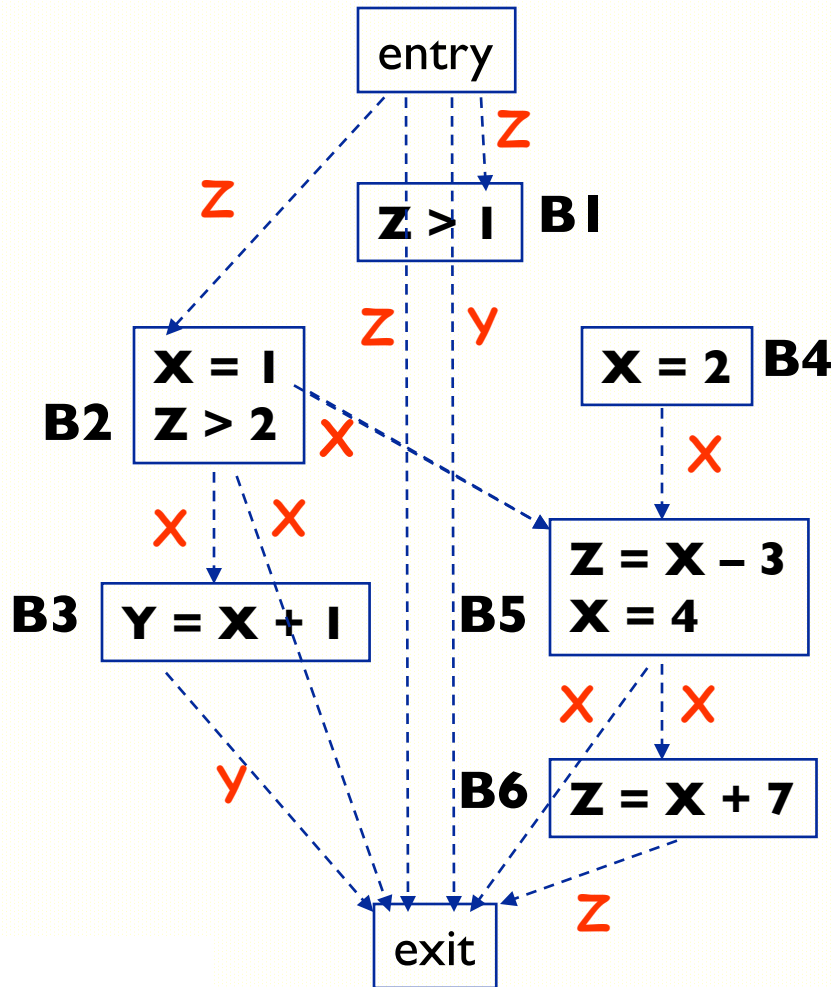
# Data-Dependence Graph (DDG)

*DDG: one node for every basic block and one edge representing the flow of data between two nodes*

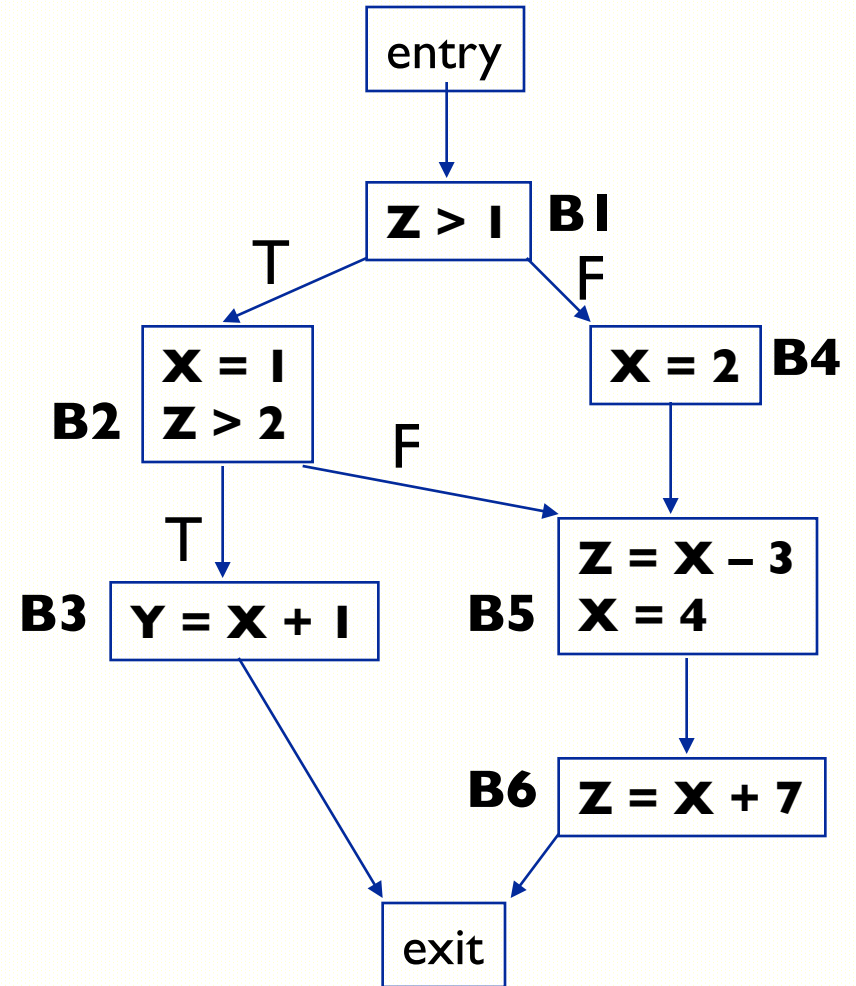


# Data-Dependence Graph

DDG



CFG

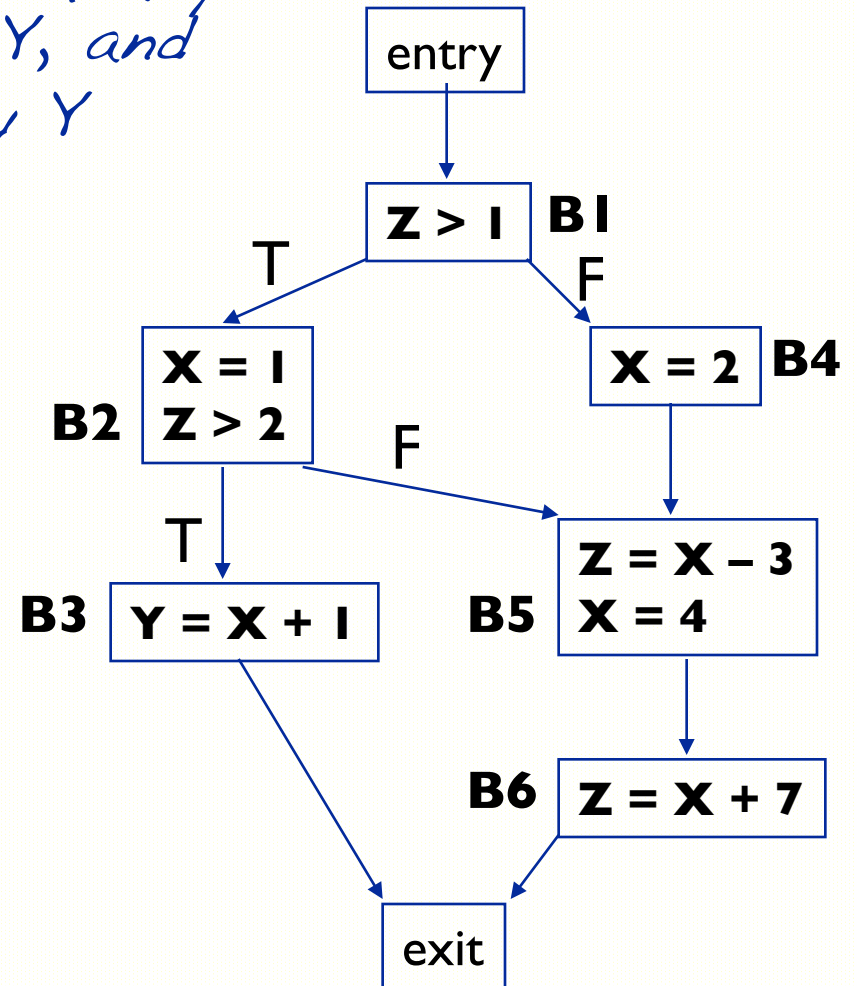


# Control-Dependence Graph (CDG)

*Y control-dependent on X iff*

1.  $\exists$  path  $P$  from  $X$  to  $Y$  with any  $Z$  in  $P$  postdominated by  $Y$ , and
2.  $X$  is not postdominated by  $Y$

*CFG*



# Control-Dependence Graph (CDG)

Intuitively: two edges out of  $X$ ; traversing one edge always leads to  $Y$ , the other may not lead to  $Y$

Dependences:

$B_1$ , exit - entry

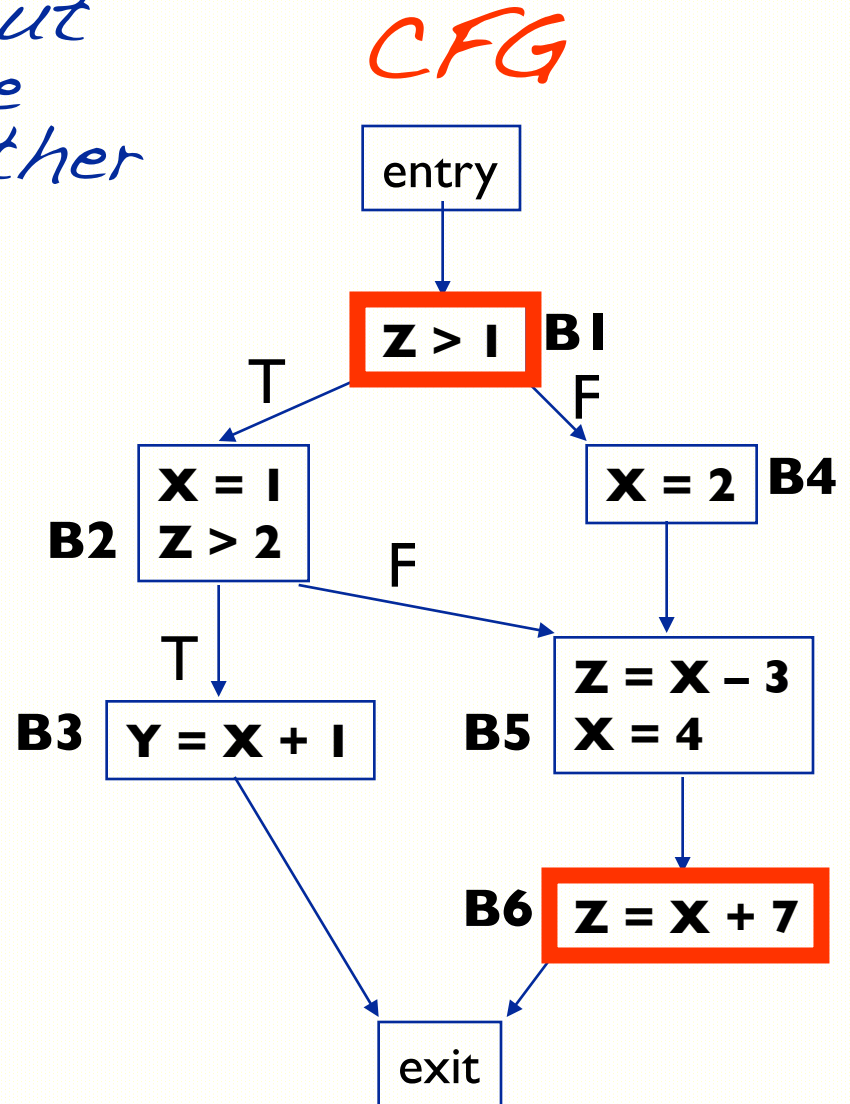
$B_2$  -  $B_1T$

$B_3$  -  $B_2T$

$B_4$  -  $B_1F$

$B_5$  -  $B_2F, B_1F$

$B_6$  -  $B_2F, B_1F$



# Program-Dependence Graph (PDG)

- A **PDG** for a program  $P$  is the combination of the DDG and CDG for  $P$
- A **PDG** contains nodes representing statements/basic blocks in  $P$  and edges representing either control or data dependence between nodes



# Static VS Dynamic Analysis

- **Static analysis** operates on a model of the SW (w/o executing it)
  - Can produce definitive information that holds for all inputs
- **Dynamic analysis** operates on dynamic information collected by running the SW
  - Produces "sampling information" that holds for the inputs considered
- **Combined static and dynamic analyses** leverage complementary strengths

# Do We Need Dynamic Analysis?

- Imprecision of static analysis
- Need to test for properties of executions (e.g., debugging)
- Need to test assumptions about the environment
- Need to determine performance for the average case
- Need to test for non-functional properties, such as usability
- ...

# Examples of Dynamic Analysis

- Testing
- Profiling
- Coverage analysis
- Dynamic-invariant detection
- Assertions
- Dynamic tainting
- Dynamic slicing
- ...

# Issues in Dynamic Analysis

- Collecting dynamic data
  - Instrumentation
  - Runtime system
  - Debugging interfaces
- Overhead
- Making sure observations don't change the behavior of the system
- Selecting the "right" inputs

# SOFTWARE TESTING

## GENERAL CONCEPTS

# Software Is Buggy

- On average, 1-5 errors per 1KLOC
- Windows 2000
  - 35M LOC
  - 63,000 known bugs at the time of release
  - 2 per 1,000 lines
- For mass market software 100% correct is infeasible, but
- We must *verify* the SW as much as possible

# Failure, Fault, Error

## Failure

Observable incorrect behavior of a program. Conceptually related to the behavior of the program, rather than its code.

## Fault (bug)

Related to the code. Necessary (not sufficient!) condition for the occurrence of a failure.

## Error

Cause of a fault. Usually a human error (conceptual, typo, etc.)

# Failure, Fault, Error: Example

```
1. int double(int param) {  
2.     int result;  
3.     result = param * param;  
4.     return(result);  
5. }
```

A call to `double(3)` returns 9

Result 9 represents a **failure**

Such failure is due to the **fault** at line 3

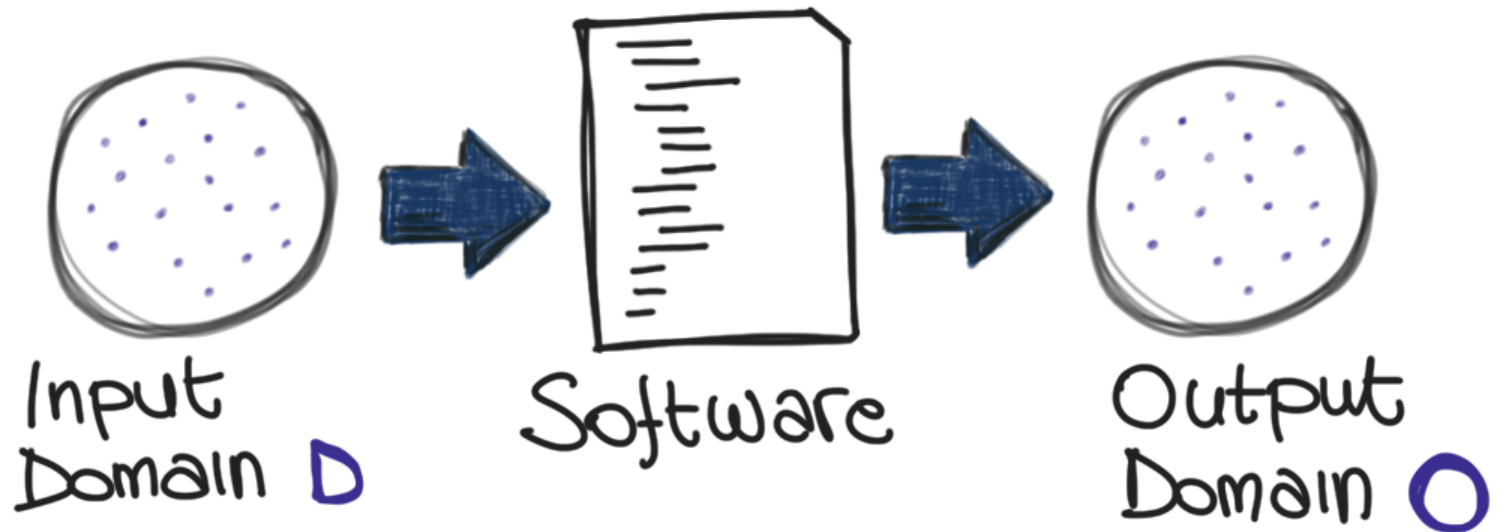
The **error** is a typo (hopefully)



# Approaches to Verification

- *Testing*: exercising software to try and generate failures
- *Static verification*: identify (specific) problems statically, that is, considering all possible executions
- *Inspection*/review/walkthrough: systematic group review of program text to detect faults
- *Formal proof*: proving that the program text implements the program specification

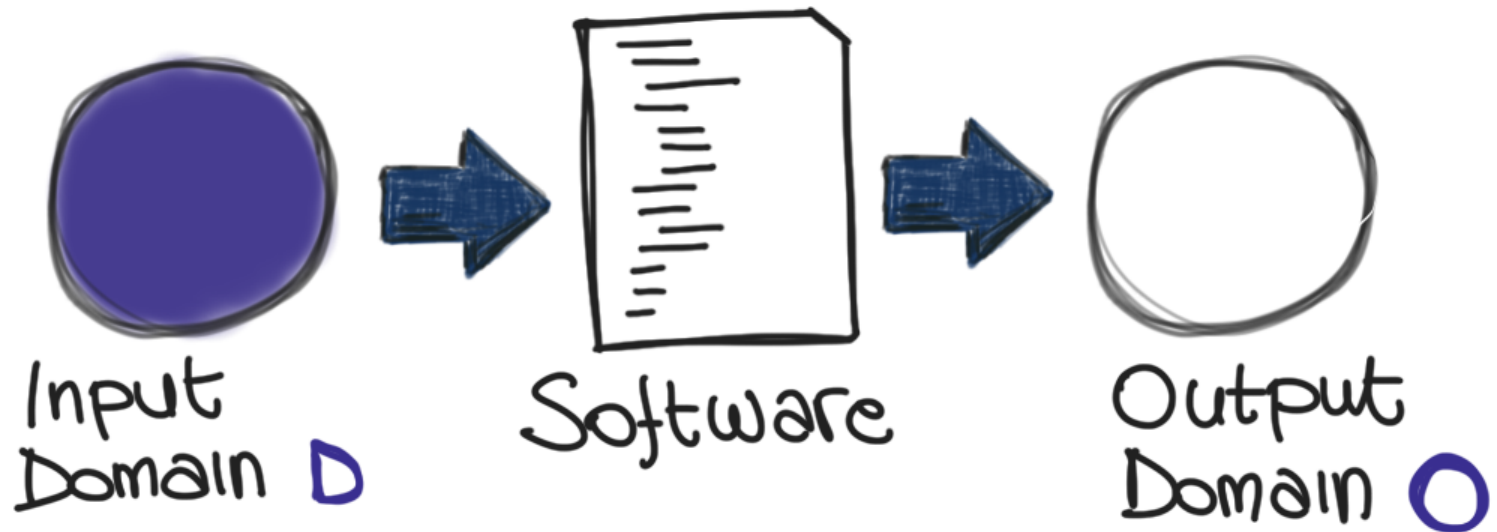
# TESTING



Test case :  $\{i \in D, o \in O\}$

Test suite : set of test cases

# VERIFICATION

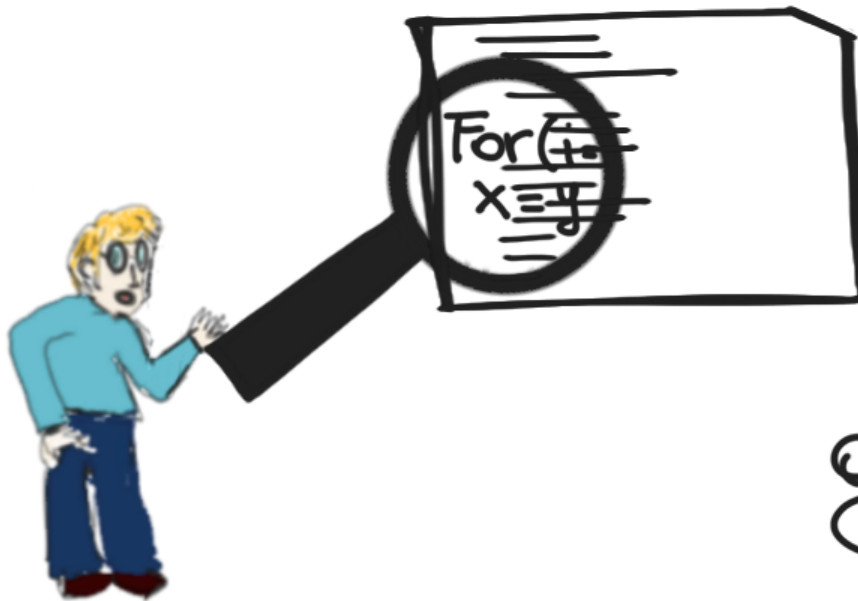


Considers all possible  
inputs (executions)

# INSPECTIONS

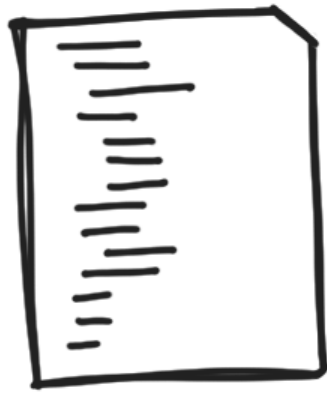
(AKA

- reviews
- walkthroughs)

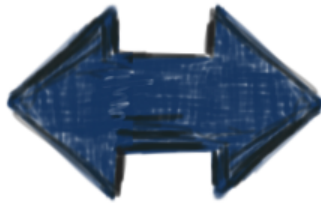


Manual  
group activity

# FORMAL PROOF (OF CORRECTNESS)



Program



Specification

Given a formal specification, checks that the code corresponds to such specification

# Comparison

## Testing

- Pros: no false positives
- Limits: incomplete

## Static verification

- P: complete (consider all program behaviors)
- L: false positives, expensive

## Inspection

- P: systematic, thorough
- L: informal, subjective

## Formal proof (of correctness)

- P: strong guarantees
- L: complex, expensive (requires a spec)



# TODAY, QA IS MOSTLY TESTING

"50% of my company employees are Testers,  
and The rest spends 50% of Their Time Testing"  
Who said That?

# What is Testing?

Testing == To execute a program with a sample of the input data

- Dynamic technique: program must be executed
- Optimistic approximation:
  - The program under test is exercised with a (very small) subset of all the possible input data
  - We assume that the behavior with any other input is consistent with the behavior shown for the selected subset of input data



# Testing Techniques

There are a number of techniques

- Different processes
- Different artifacts
- Different approaches

There are no perfect techniques

- Testing is a best-effort activity

There is no best technique

- Different contexts
- Complementary strengths and weaknesses
- Trade-offs

# TESTING GRANULARITY LEVELS



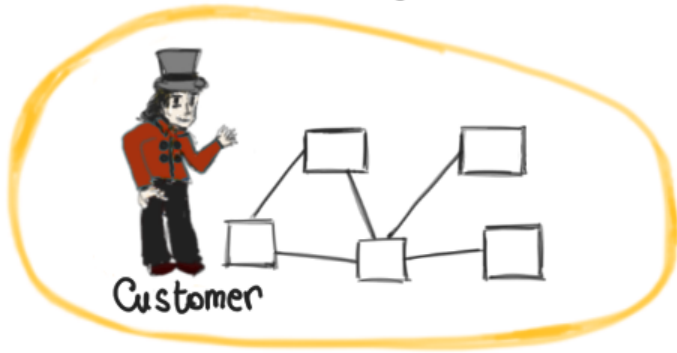
Unit Testing



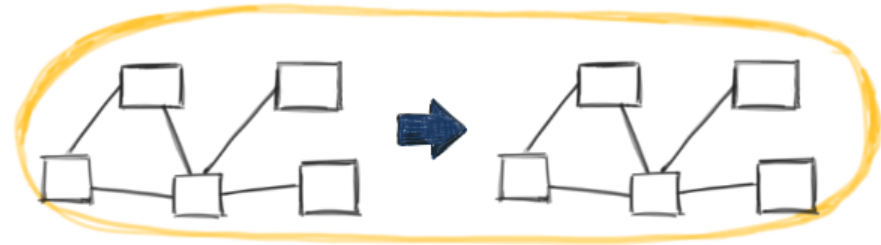
Integration Testing



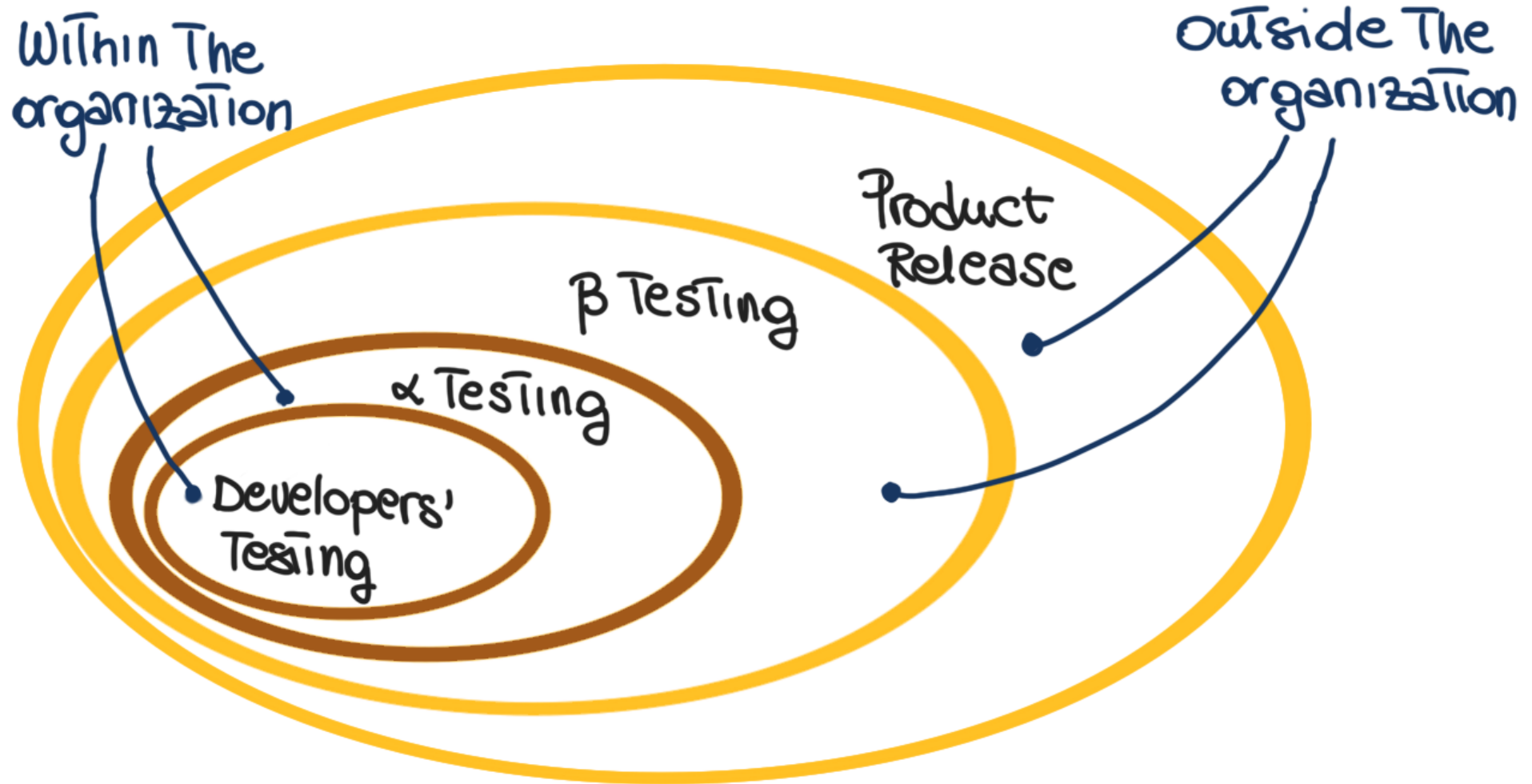
System Testing



Acceptance Testing



Regression Testing



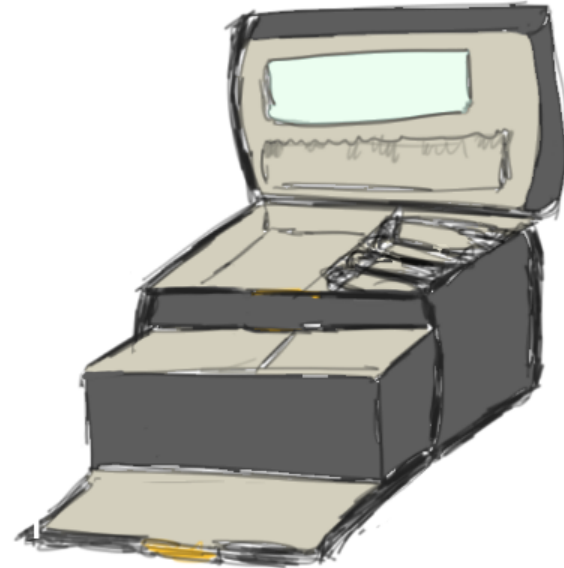
# *Functional vs. Structural Testing*





## BLACK-BOX TESTING

- based on a description of the software (specification)
- cover as much specified behavior as possible
- cannot reveal errors due to implementation details



## WHITE-BOX TESTING

- based on the code
- cover as much coded behavior as possible
- cannot reveal errors due to missing paths

# BLACK-BOX TESTING EXAMPLE

Specification: inputs an integer and prints it

# BLACK-BOX TESTING EXAMPLE

Specification: inputs an integer and prints it

```
1. void printNumBytes ( param )  
2.   if (param < 1024) printf("%.d", param);  
3.   else printf("%.d KB", param/1024);  
4. }
```



# WHITE-BOX TESTING EXAMPLE

```
1. int fun(int param){  
2.     int result;  
3.     result = param/2;  
4.     return result;  
5. }
```



# WHITE-BOX TESTING EXAMPLE

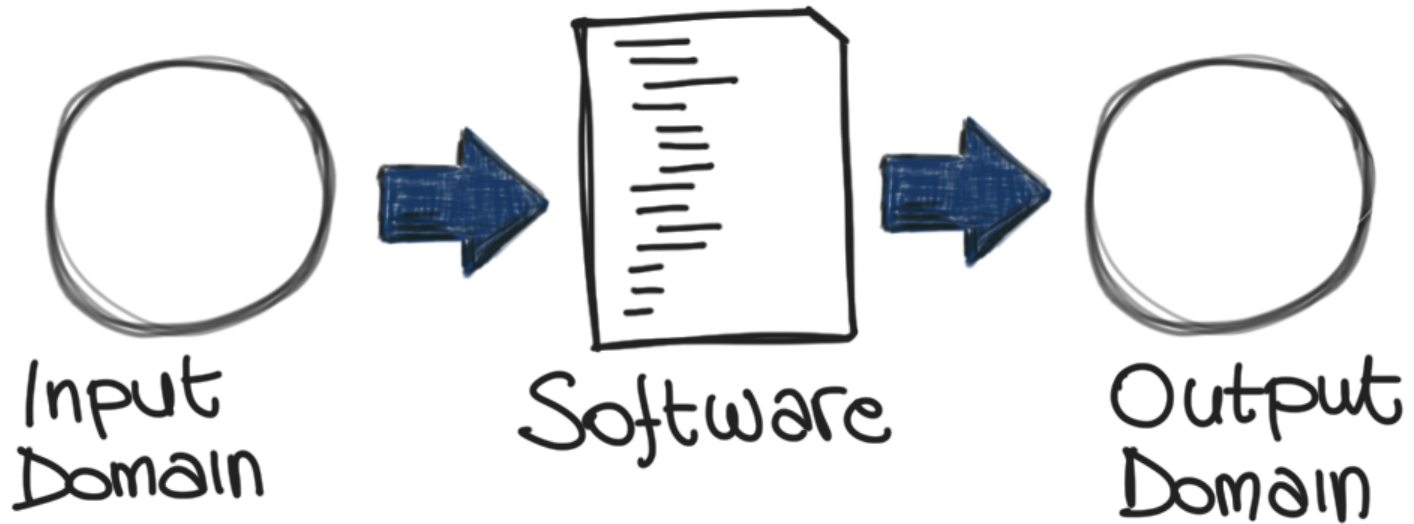
Specification: inputs an integer param and returns  
half of its value if even, its value otherwise

```
1. int fun(int param){  
2.     int result;  
3.     result = param/2;  
4.     return result;  
5. }
```

# SOFTWARE TESTING

*Test-Data Selection*

# TEST DATA SELECTION



# STRAW-MAN IDEA : EXHAUSTIVE TESTING !



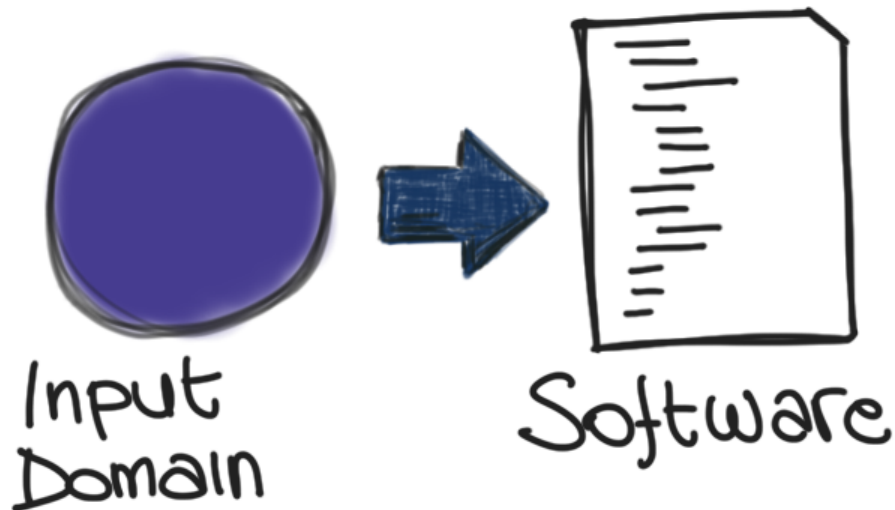
# STRAW-MAN IDEA : EXHAUSTIVE TESTING !



Considers all possible  
inputs (executions)



# STRAW-MAN IDEA : EXHAUSTIVE TESTING !



How long would it  
take to exhaustively  
test the function

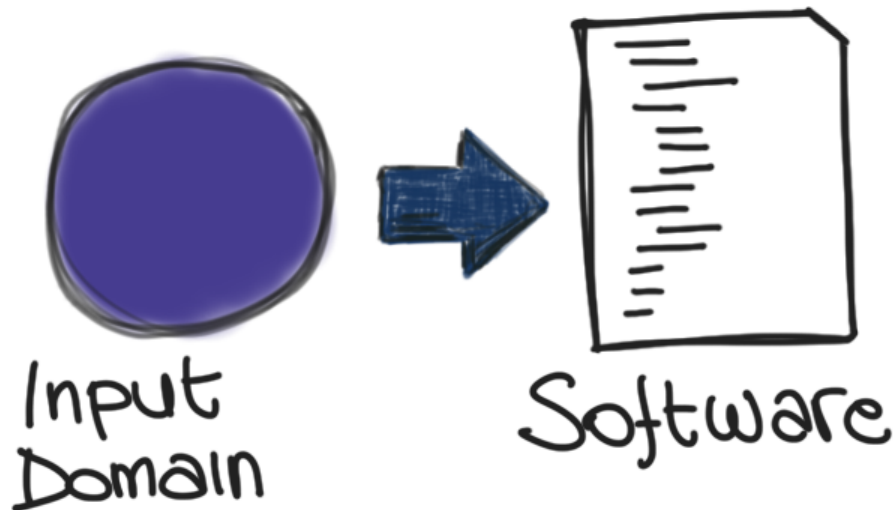
`print Sum(int a, int b)`?

[

]



# STRAW-MAN IDEA : EXHAUSTIVE TESTING !



How long would it  
take to exhaustively  
test the function

`print Sum(int a, int b)`?

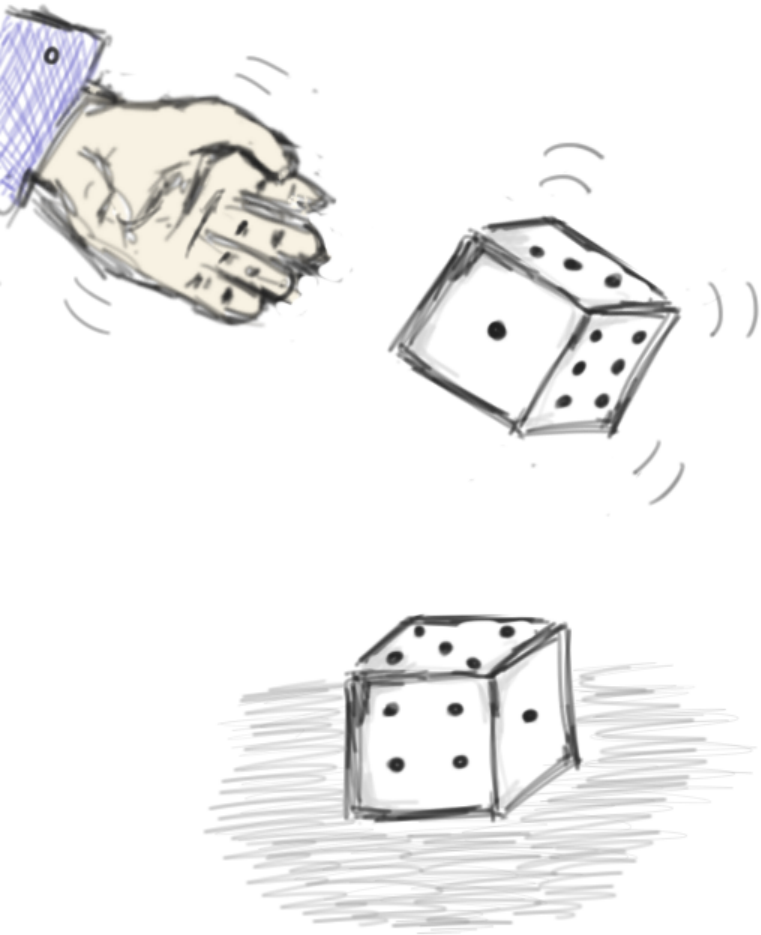
$$2^{32} \times 2^{32} = 2^{64} \approx 10^{19} \text{ tests}$$

1 test per nanosecond ( $10^9$  tests/sec)

$\Rightarrow 10^{10}$  seconds

[ ~600 years ]

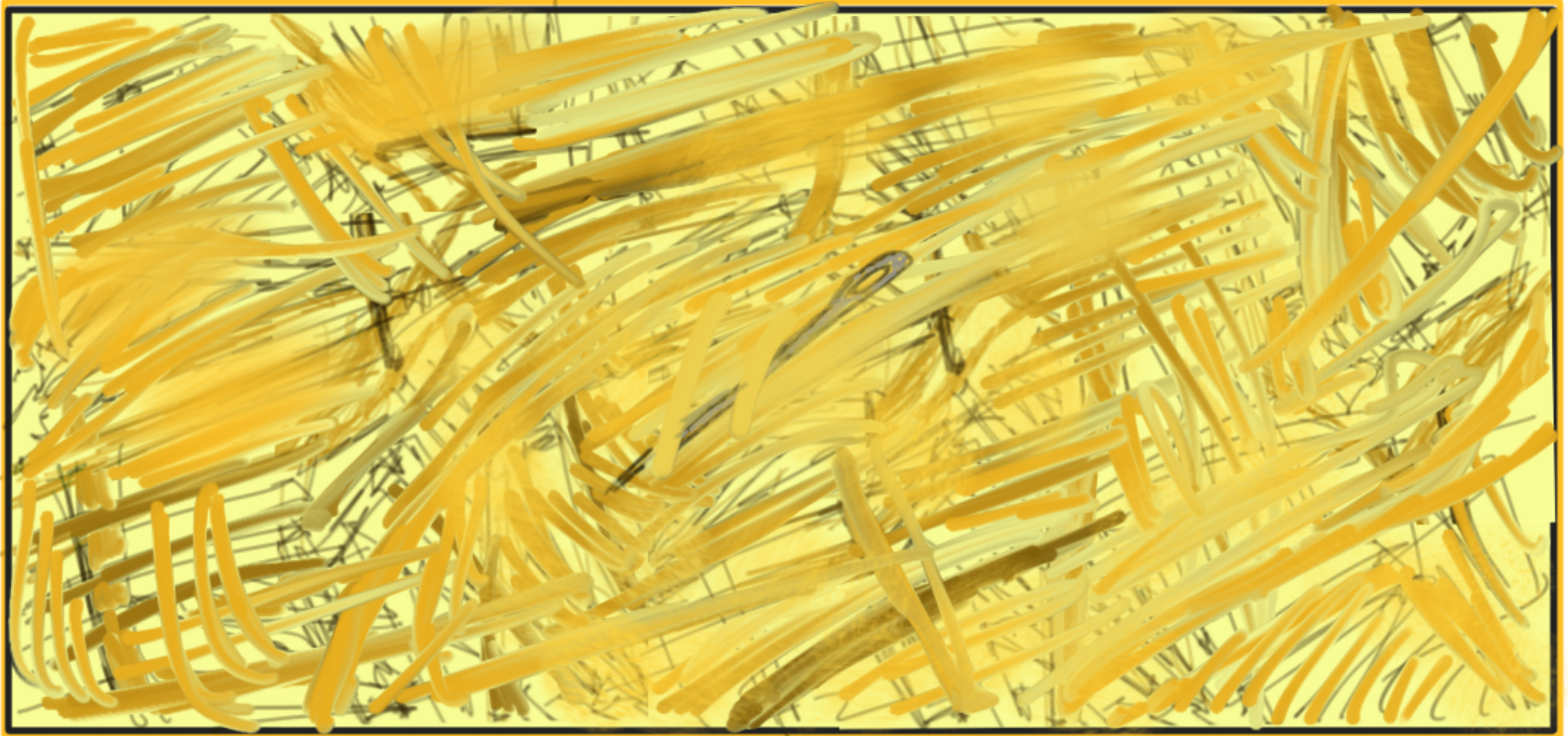
# RANDOM TESTING



- pick inputs uniformly
- all inputs considered equal
- no designer bias



SO WHY NOT RANDOM ?



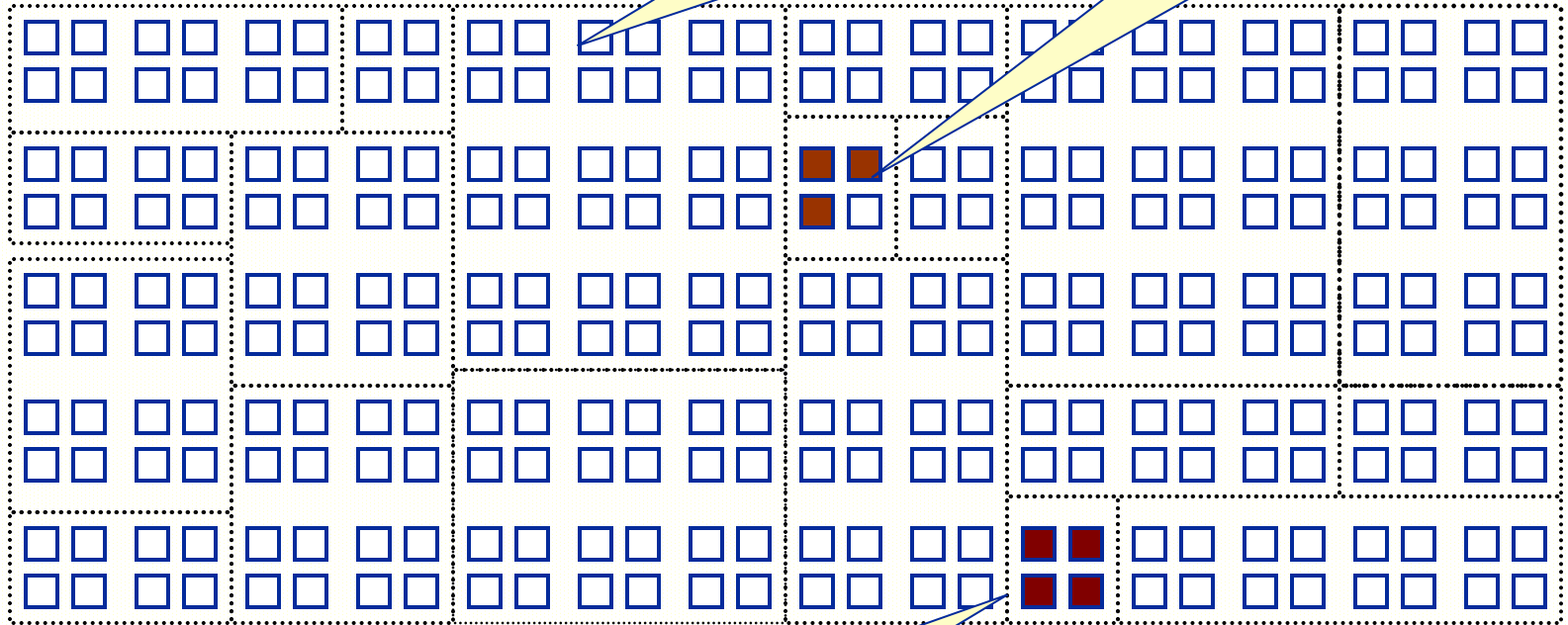
# Systematic Partition Testing

The space of possible input values  
(the haystack)

- Failure (valuable test case)
- No failure

Failures are sparse  
in the space of  
possible inputs ...

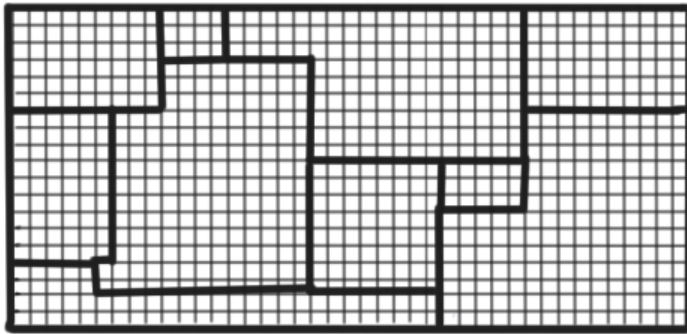
... but dense in  
some parts of the  
space



If we systematically test some  
cases from each part, we will  
include the dense parts

Functional testing is one way  
of drawing lines to isolate  
regions with likely failures

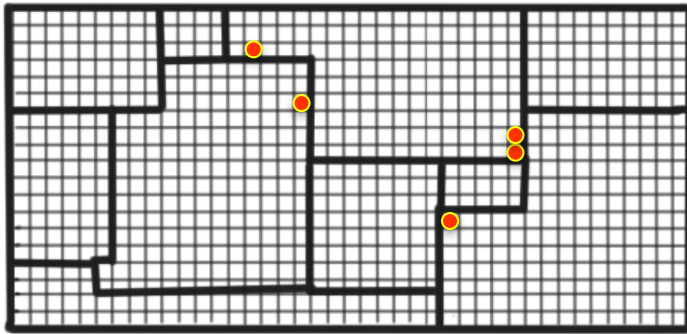
# BOUNDARY VALUES



## Basic idea

Errors tend to occur at the boundary of a (sub)domain

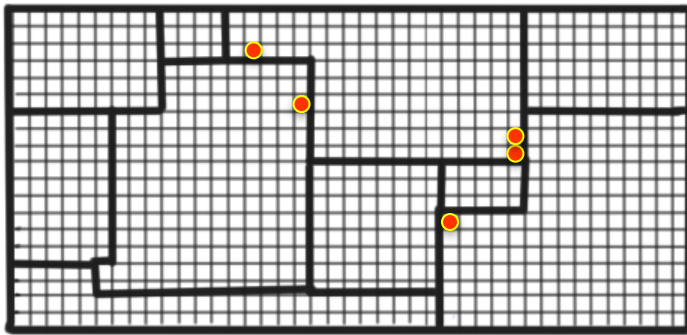
# BOUNDARY VALUES



Basic idea

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## BOUNDARY VALUES

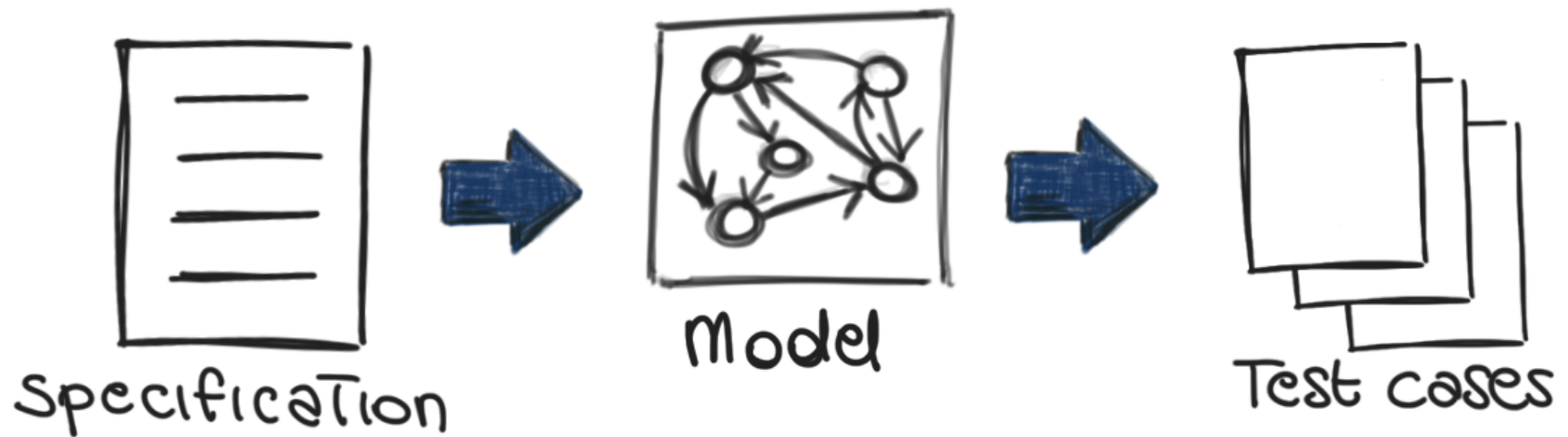


## BASIC idea

Errors tend to occur at the boundary of a (sub)domain

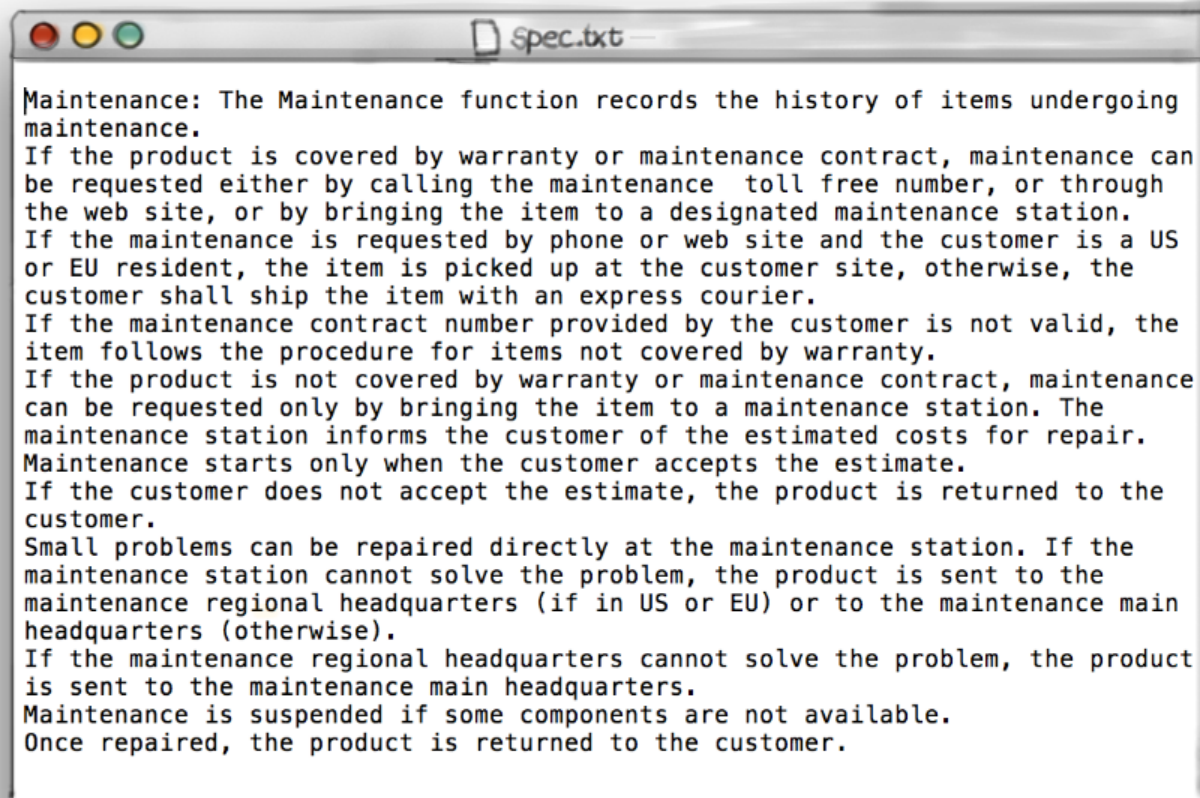
⇒ Select inputs at these boundaries

# MODEL-BASED TESTING

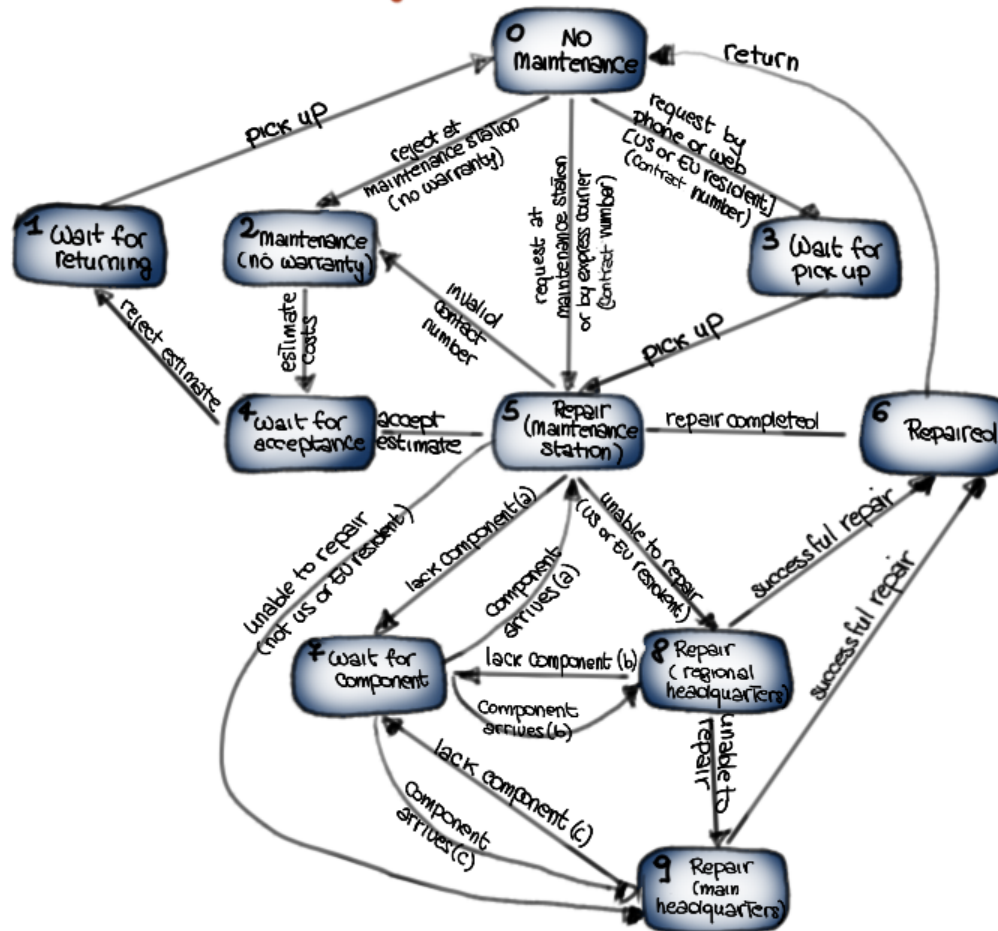




# FROM AN INFORMAL SPECIFICATION...

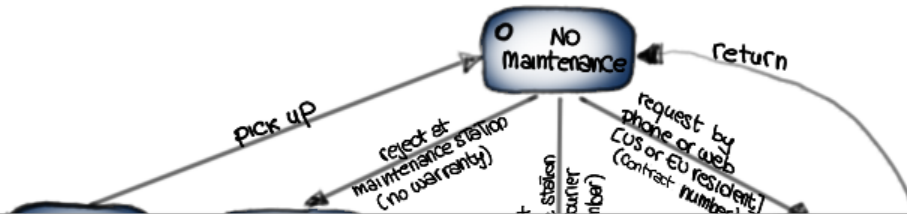


# ... TO A FINITE STATE MACHINE

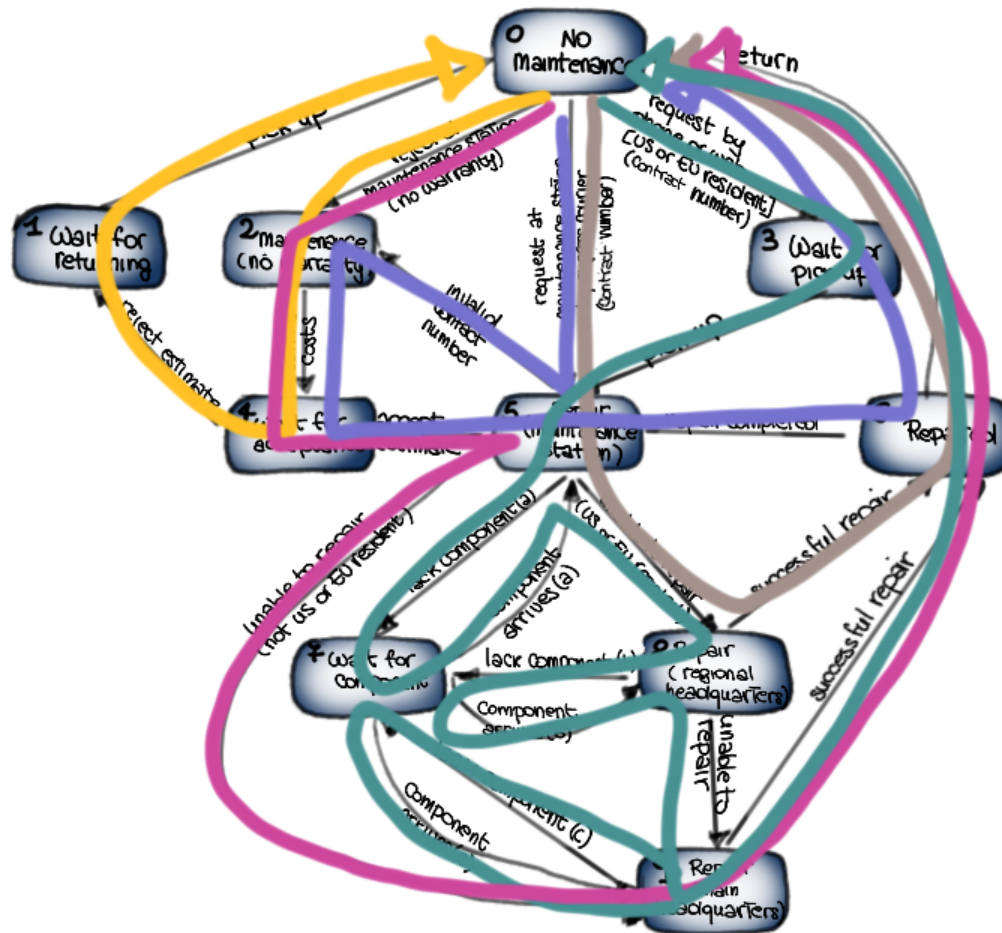




# TO A SET OF TEST CASES



# TO A SET OF TEST CASES



**Maintenance:** The Maintenance function records the history of items undergoing maintenance.

If the product is covered by warranty or maintenance contract, maintenance can be requested either by calling the maintenance toll free number, or through the web site, or by bringing the item to a designated maintenance station.

If the maintenance is requested by phone or web site and the customer is a US or EU resident, the item is picked up at the customer site, otherwise, the customer shall ship the item with an express courier.

If the maintenance contract number provided by the customer is not valid, the item follows the procedure for items not covered by warranty.

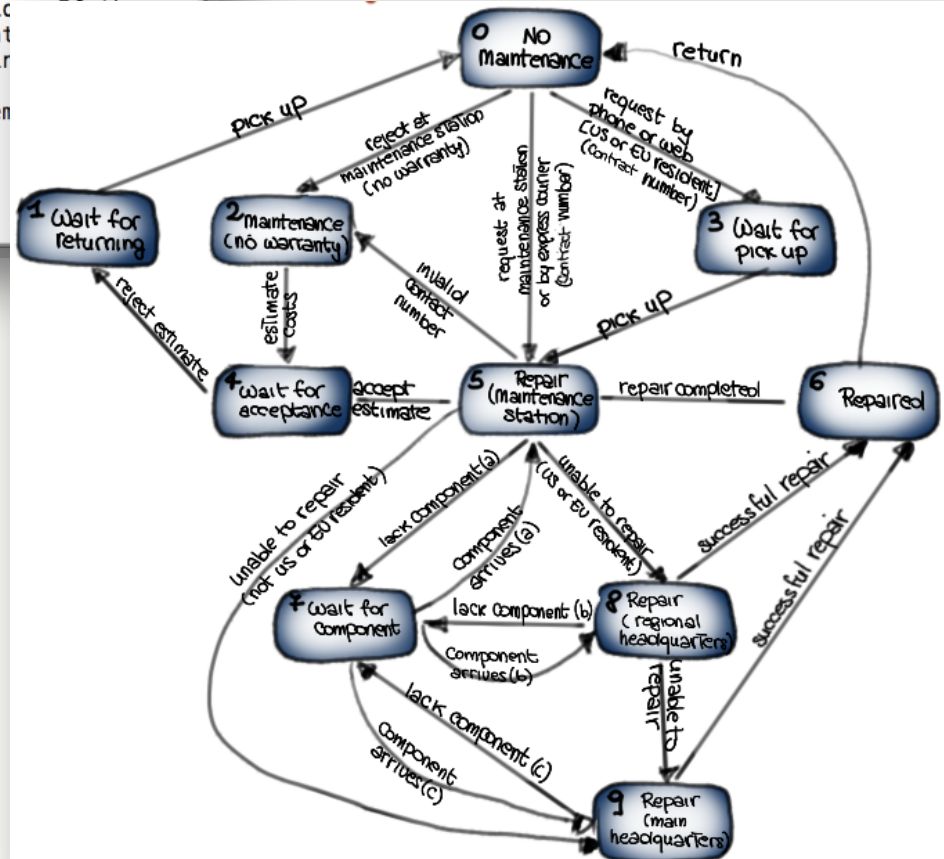
If the product is not covered by warranty or maintenance contract, maintenance can be requested only by bringing the item to a maintenance station. The maintenance station informs the customer of the estimated costs for repair. Maintenance starts only when the customer accepts the estimate.

If the customer does not accept the estimate, the product is returned to the customer.

Small problems can be repaired directly at the maintenance station. If the maintenance station cannot solve the problem, the product is sent to maintenance regional headquarters (if in US or EU) or to the main headquarters (otherwise).

If the maintenance regional headquarters cannot solve the problem is sent to the maintenance main headquarters.

Maintenance is suspended if some components are not available. Once repaired, the product is returned to the customer.



# SOME CONSIDERATIONS

## Applicability

- very general approach
- in UML, state machine are readily available

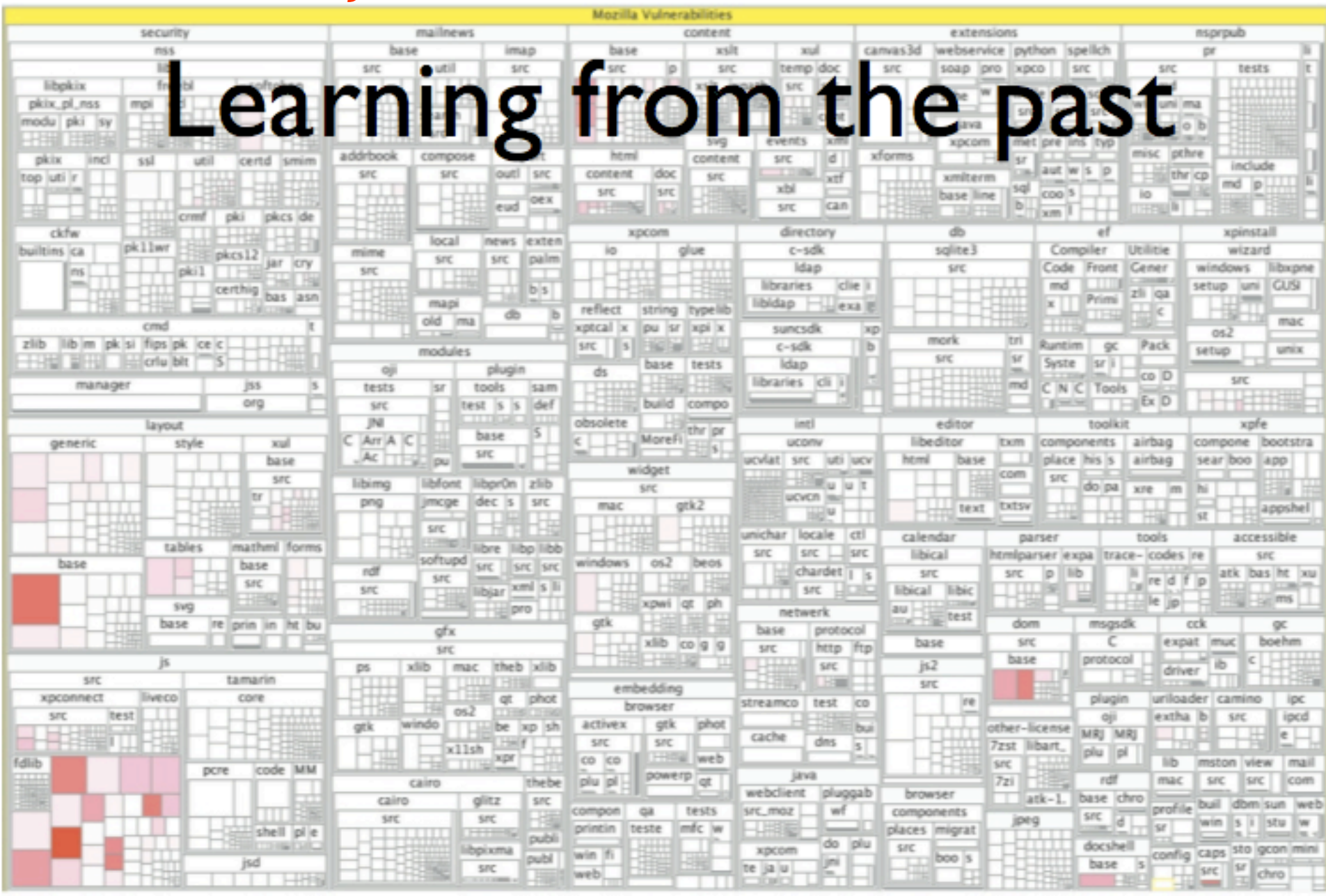
Abstraction is key

many other approaches

- decision tables
- flow graphs
- historical models
- ...



# Learning from the past



# Historical models

Le

## Pareto's Law

st

Approximately 80% of defects  
come from 20% of modules

# COVERAGE CRITERIA

Defined in terms of  
test requirements

Result in  
test specifications  
test cases

# printSum: *test requirements*

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
}
```

**req #1**

**req #2**



# printSum: *test specifications*

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
}
```

**$a + b > 0$**

**$a + b < 0$**



## printSum: *test cases*

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
}
```

Test Spec #1  
 **$a + b > 0$**

Test Spec #2  
 **$a + b < 0$**

#1 ((a=[**3**], b=[**9**]), (output color=[**red**], output value=[**12**]))  
#2 ((a=[**0**], b=[**-1**]), (output color=[**blue**], output value=[**-1**]))

# STATEMENT COVERAGE

Test  
requirements

Coverage  
measure

# STATEMENT COVERAGE

Test  
requirements

statements in the program

Coverage  
measure

# STATEMENT COVERAGE

Test  
requirements

statements in the program

Coverage  
measure

$$\frac{\text{number of executed statements}}{\text{total number of statements}}$$

# printSum: *statement coverage*

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
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    else if (result < 0)  
        printcol("blue", result);  
}
```

# printSum: *statement coverage*

**a == 3**

**b == 9**

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
}
```

**Coverage: 0%**

# printSum: *statement coverage*

```
a == 3  
b == 9
```

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
}
```

**Coverage: 71%**



# printSum: *statement coverage*

**a == 3**  
**b == 9**

**a == 0**  
**b == -1**

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
}
```

**Coverage: 71%**

# printSum: *statement coverage*

**a == 3**  
**b == 9**

**a == 0**  
**b == -1**

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
}
```

**Coverage: 100%**

# STATEMENT COVERAGE IN PRACTICE

Most used in industry

"Typical coverage" target is 80-90%.



Why don't we aim at 100%.

[

]

# printSum: *statement coverage*

**a == 3**  
**b == 9**

**a == 0**  
**b == -1**

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
}
```

**Coverage: 100%**

# printSum: *statement coverage*

**a == 3**  
**b == 9**

**a == 0**  
**b == -1**

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
    [else do nothing]  
}
```

**Coverage: 100%**

# printSum: *statement coverage*

**a == 3**  
**b == 9**

**a == 0**  
**b == -1**

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
    [else do nothing]  
}
```

**Coverage: 100%**

# BRANCH COVERAGE

Test  
requirements

Coverage  
measure

# BRANCH COVERAGE

Test  
requirements

branches in the program

Coverage  
measure



# BRANCH COVERAGE

Test  
requirements

branches in the program

Coverage  
measure

$$\frac{\text{number of executed branches}}{\text{total number of branches}}$$

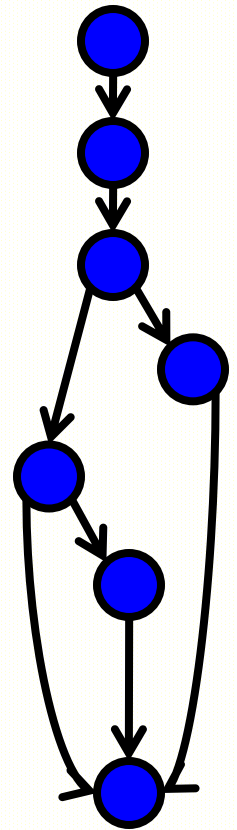
# printSum: *branch coverage*

**a == 3**  
**b == 9**

**a == 0**  
**b == -1**

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
    [else do nothing]}  
  

```



**Coverage: ?**

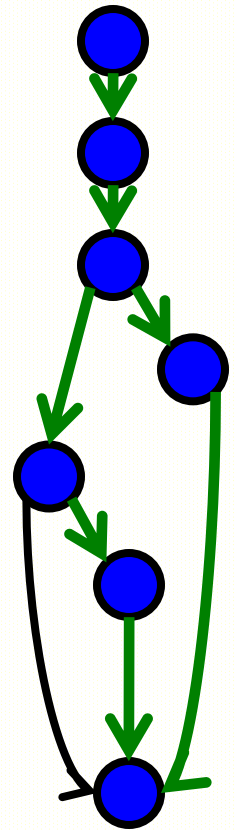
# printSum: *branch coverage*

**a == 3**  
**b == 9**

**a == 0**  
**b == -1**

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
    [else do nothing]}  
  

```



**Coverage: 75%**

# printSum: *branch coverage*

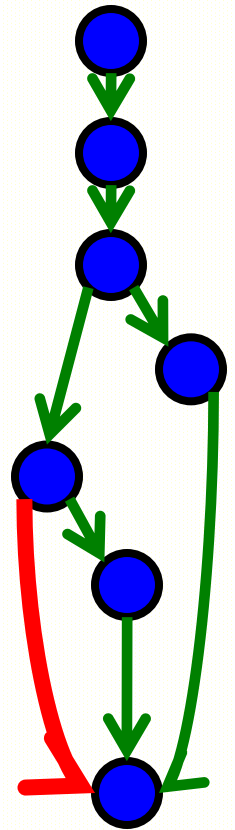
**a == 3  
b == 9**

**a == 0  
b == -1**

**a == -5  
b == 5**

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
    [else do nothing]}  
  

```



**Coverage: 75%**

# printSum: *branch coverage*

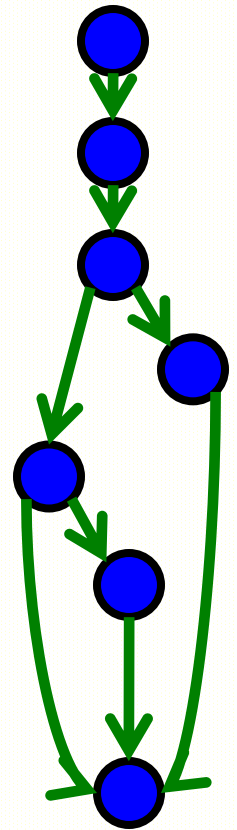
**a == 3  
b == 9**

**a == 0  
b == -1**

**a == -5  
b == 5**

```
printSum(int a, int b) {  
    int result = a + b;  
    if (result > 0)  
        printcol("red", result);  
    else if (result < 0)  
        printcol("blue", result);  
    [else do nothing]}  
  

```



**Coverage: 100%**

# TEST CRITERIA SUBSUMPTION

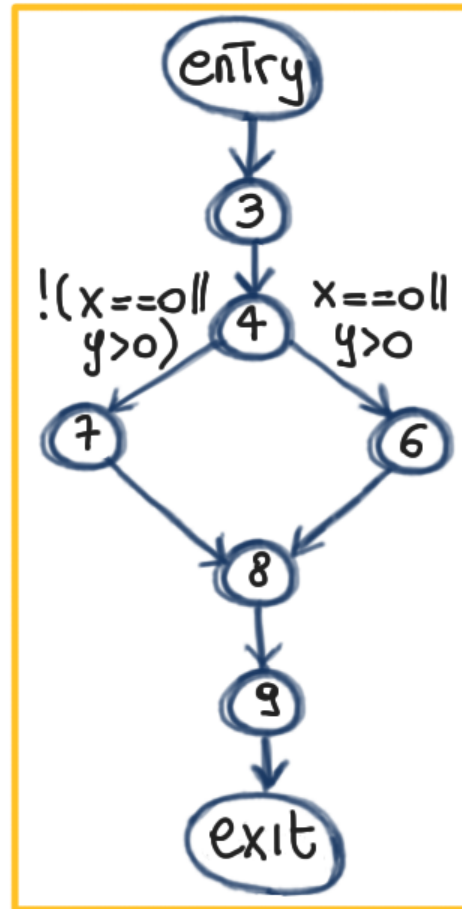


# LET'S CONSIDER ANOTHER EXAMPLE

```
1. void main(){  
2.   float x, y;  
3.   read(x);  
4.   read(y);  
5.   if ((x==0) || (y>0))  
6.       y = y/x;  
7.   else x = y+2;  
8.   write(x);  
9.   write(y);  
10. }
```

# LET'S CONSIDER ANOTHER EXAMPLE

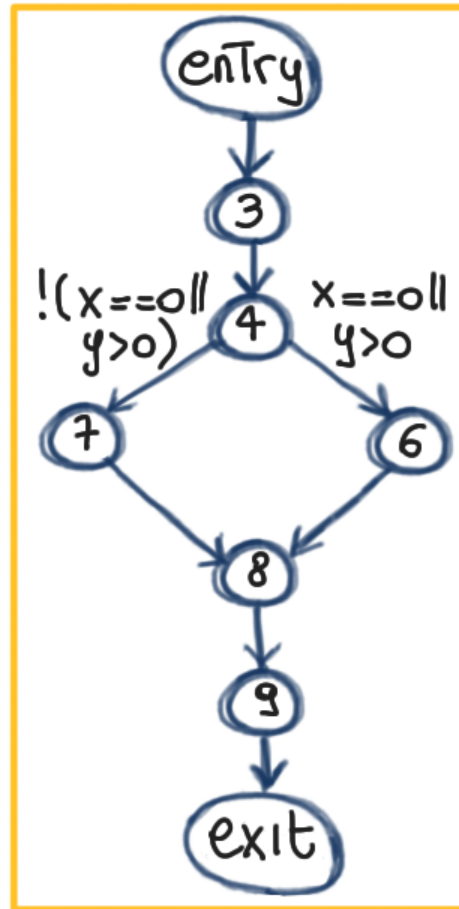
```
1. void main(){  
2.   float x, y;  
3.   read(x);  
4.   read(y);  
5.   if ((x==0) || (y>0))  
6.     y = y/x;  
7.   else x = y+2;  
8.   write(x);  
9.   write(y);  
10. }
```





# LET'S CONSIDER ANOTHER EXAMPLE

```
1. void main(){
2.   float x, y;
3.   read(x);
4.   read(y);
5.   if ((x==0) || (y>0))
6.     y = y/x;
7.   else x = y+2;
8.   write(x);
9.   write(y);
10. }
```

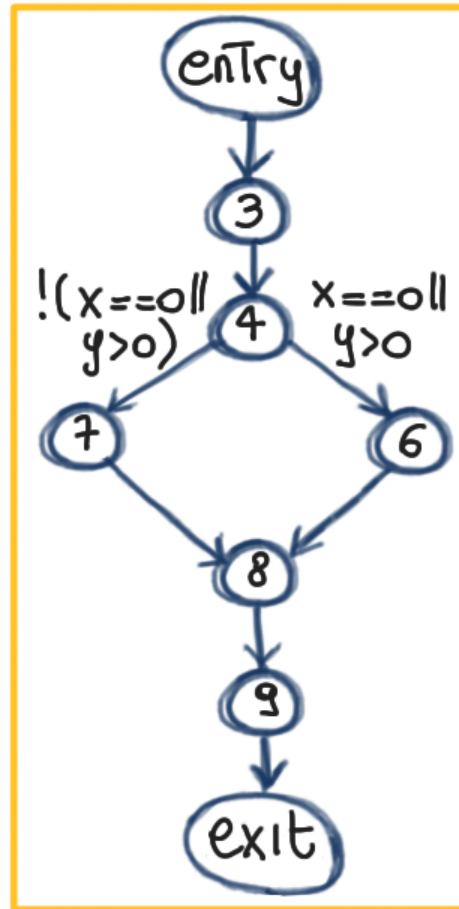


Tests: (x=5, y=6)  
(x=5, y=-5)

Branch coverage:

# LET'S CONSIDER ANOTHER EXAMPLE

```
1. void main(){
2.   float x,y;
3.   read(x);
4.   read(y);
5.   if ((x==0) || (y>0))
6.     y = y/x;
7.   else x = y+2;
8.   write(x);
9.   write(y);
10. }
```

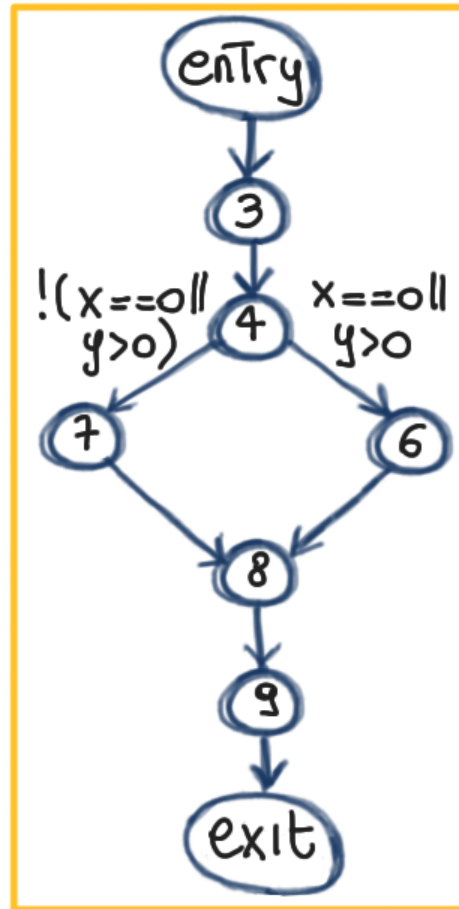


Tests: (x=5, y=6)  
(x=5, y=-5)

Branch coverage: 100%

# LET'S CONSIDER ANOTHER EXAMPLE

```
1. void main(){
2.   float x, y;
3.   read(x);
4.   read(y);
5.   if ((x==0) || (y>0))
6.     y = y/x;
7.   else x = y+2;
8.   write(x);
9.   write(y);
10. }
```



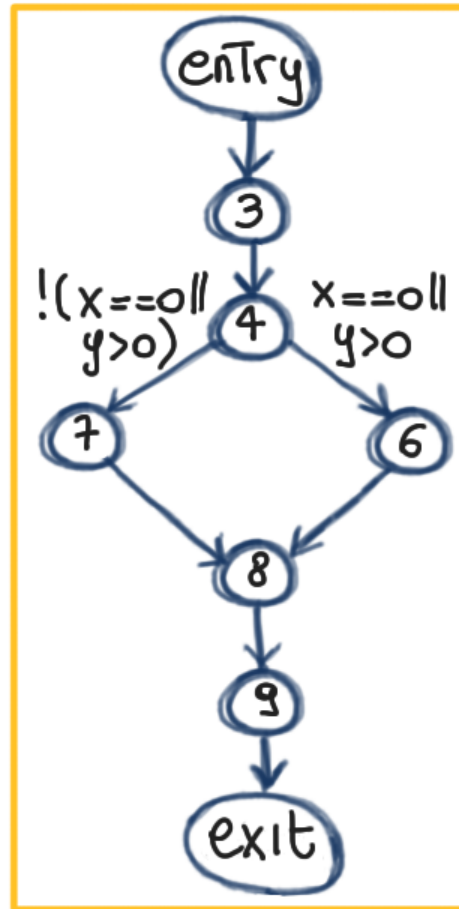
Tests: (x=5, y=6)  
(x=5, y=-5)

Branch coverage: 100%

How can we be more thorough?

# LET'S CONSIDER ANOTHER EXAMPLE

```
1. void main(){
2.   float x, y;
3.   read(x);
4.   read(y);
5.   if ((x==0) || (y>0))
6.     y = y/x;
7.   else x = y+2;
8.   write(x);
9.   write(y);
10. }
```



Tests: (x=5, y=6)  
(x=5, y=-5)

Branch coverage: 100%

How can we be more thorough?

We can make each condition T and F

# CONDITION COVERAGE

Test  
requirements

Coverage  
measure

# CONDITION COVERAGE

Test  
requirements

individual conditions in the program

Coverage  
measure

# CONDITION COVERAGE

Test  
requirements

individual conditions in the program

Coverage  
measure

$$\frac{\text{number of conditions that are both T and F}}{\text{total number of conditions}}$$

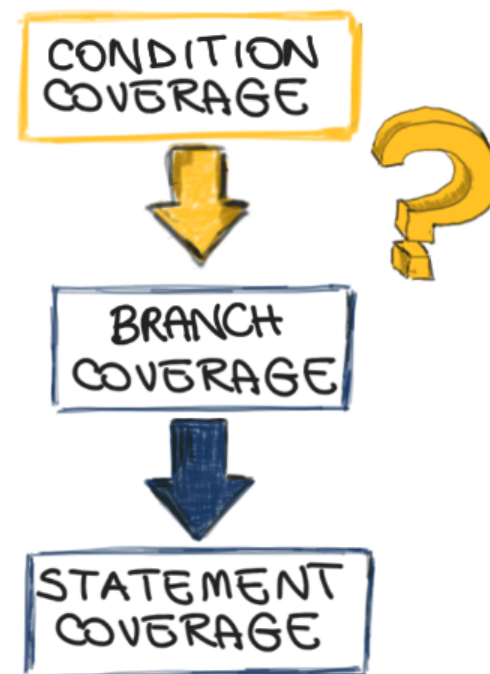


# SUBSUMPTION

Does condition coverage  
imply branch coverage?

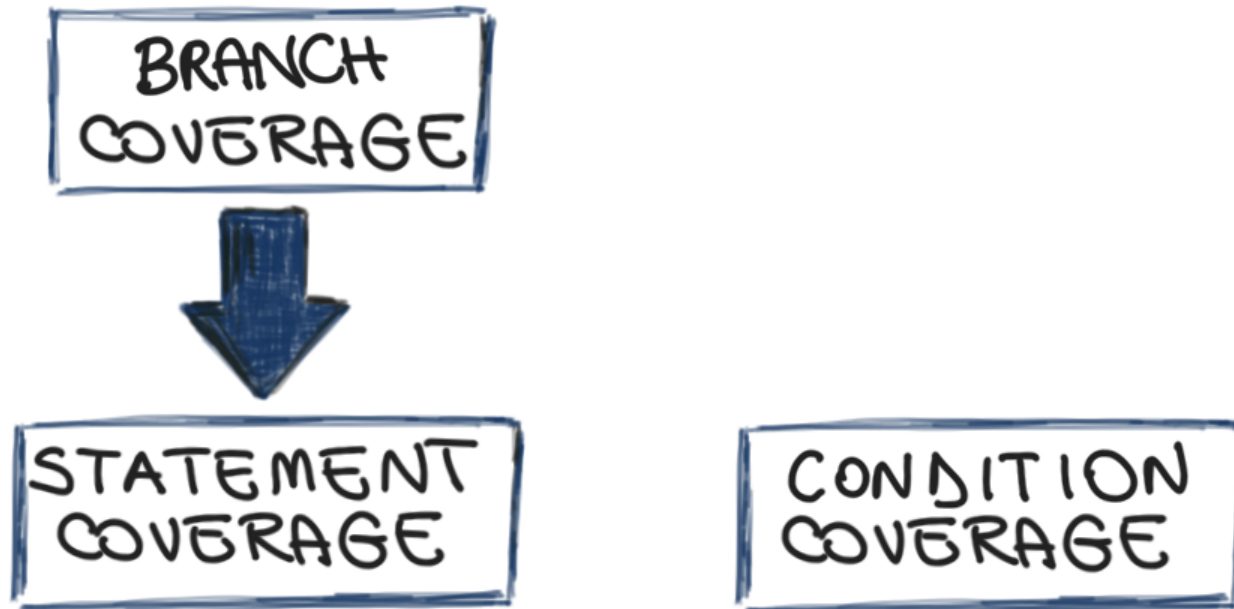
☐ Yes

☐ No



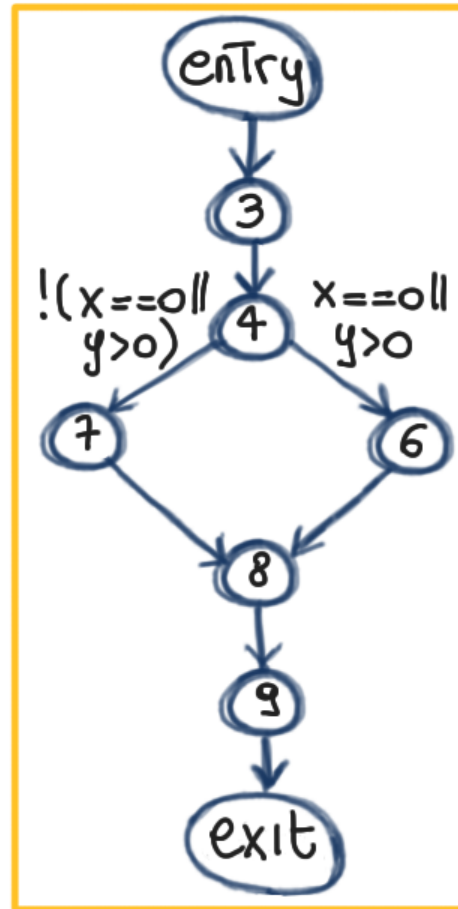


# TEST CRITERIA SUBSUMPTION



# LET'S CONSIDER OUR LAST EXAMPLE

```
1. void main(){  
2.   float x, y;  
3.   read(x);  
4.   read(y);  
5.   if ((x==0) || (y>0))  
6.     y = y/x;  
7.   else x = y+2;  
8.   write(x);  
9.   write(y);  
10. }
```



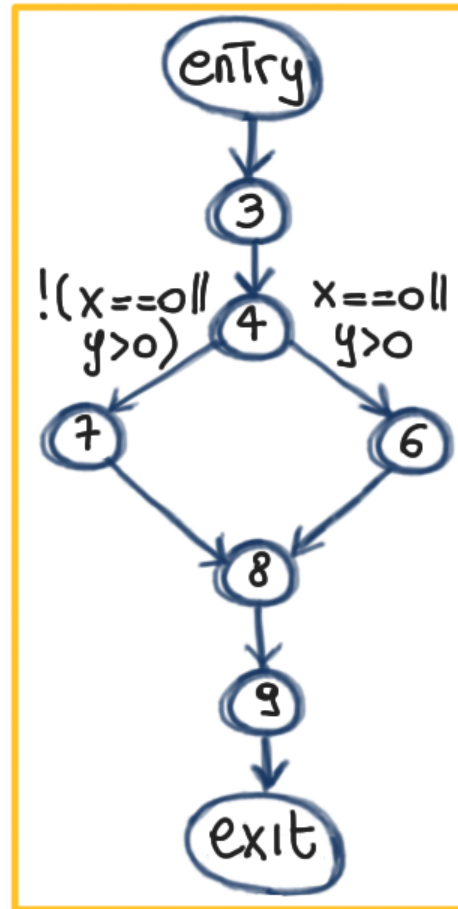
Tests:  $(x=0, y=-5)$   
 $(x=5, y=5)$

Condition coverage: 100%.

What about branch coverage?

# LET'S CONSIDER OUR LAST EXAMPLE

```
1. void main(){
2.   float x, y;
3.   read(x);
4.   read(y);
5.   if ((x==0) || (y>0))
6.     y = y/x;
7.   else x = y+2;
8.   write(x);
9.   write(y);
10. }
```



Tests:  $(x=0, y=-5)$   
 $(x=5, y=5)$

Condition coverage: 100%.

What about branch coverage? 50%.

# BRANCH AND CONDITION COVERAGE (DECISION)

Test  
requirements

branches and individual conditions  
in the program

Coverage  
measure

Computed considering both coverage  
measures



# SUBSUMPTION

Does branch and condition coverage imply branch coverage?

☐ Yes

☐ No

BRANCH AND  
CONDITION  
COVERAGE



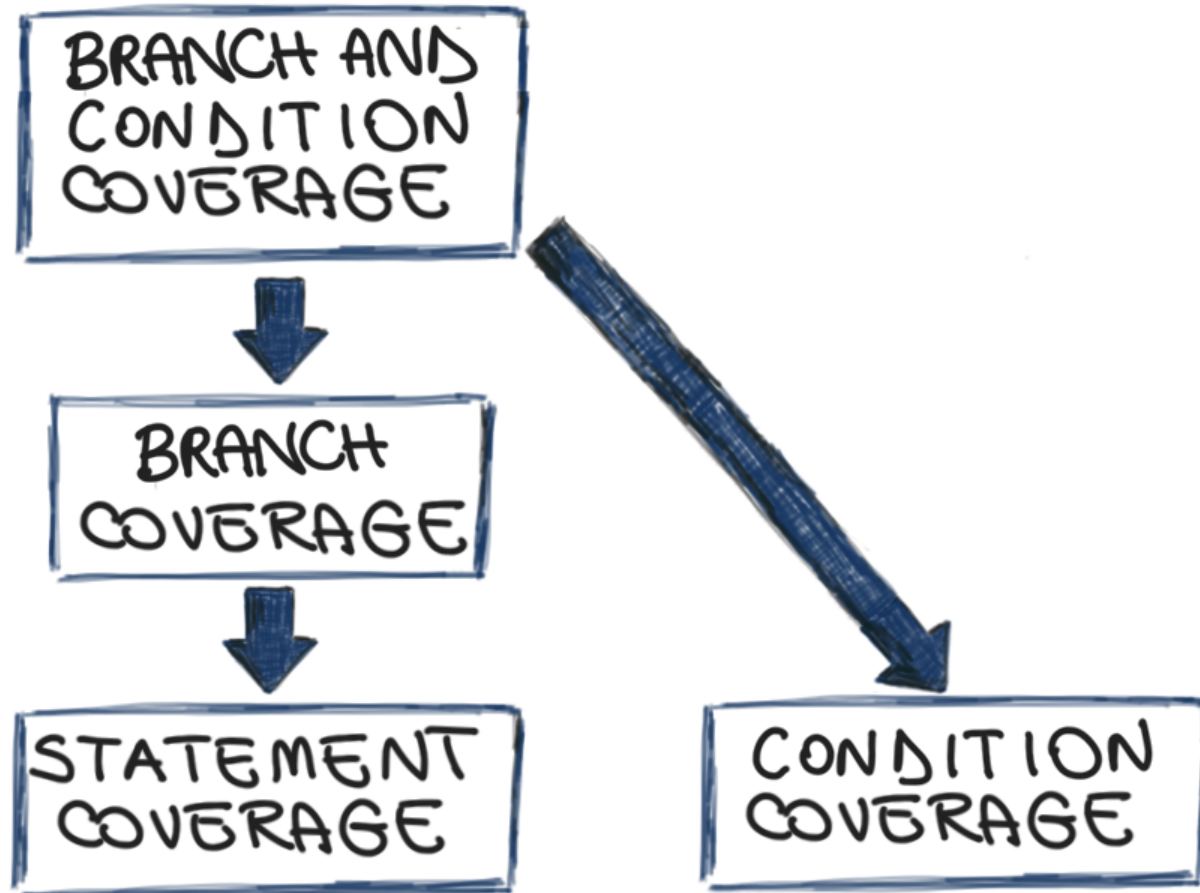
BRANCH  
COVERAGE



STATEMENT  
COVERAGE



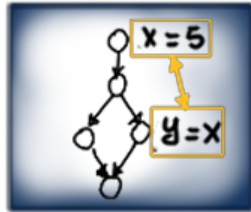
# TEST CRITERIA SUBSUMPTION



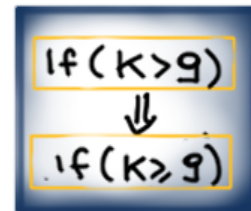
# OTHER CRITERIA



Path coverage



Data-flow coverage



Mutation coverage

# TEST CRITERIA SUBSUMPTION

Theoretical  
Criteria

PATH COVERAGE

MULTIPLE  
CONDITION  
COVERAGE

MUTATION  
COVERAGE

mc/dc

DATA-FLOW  
COVERAGE

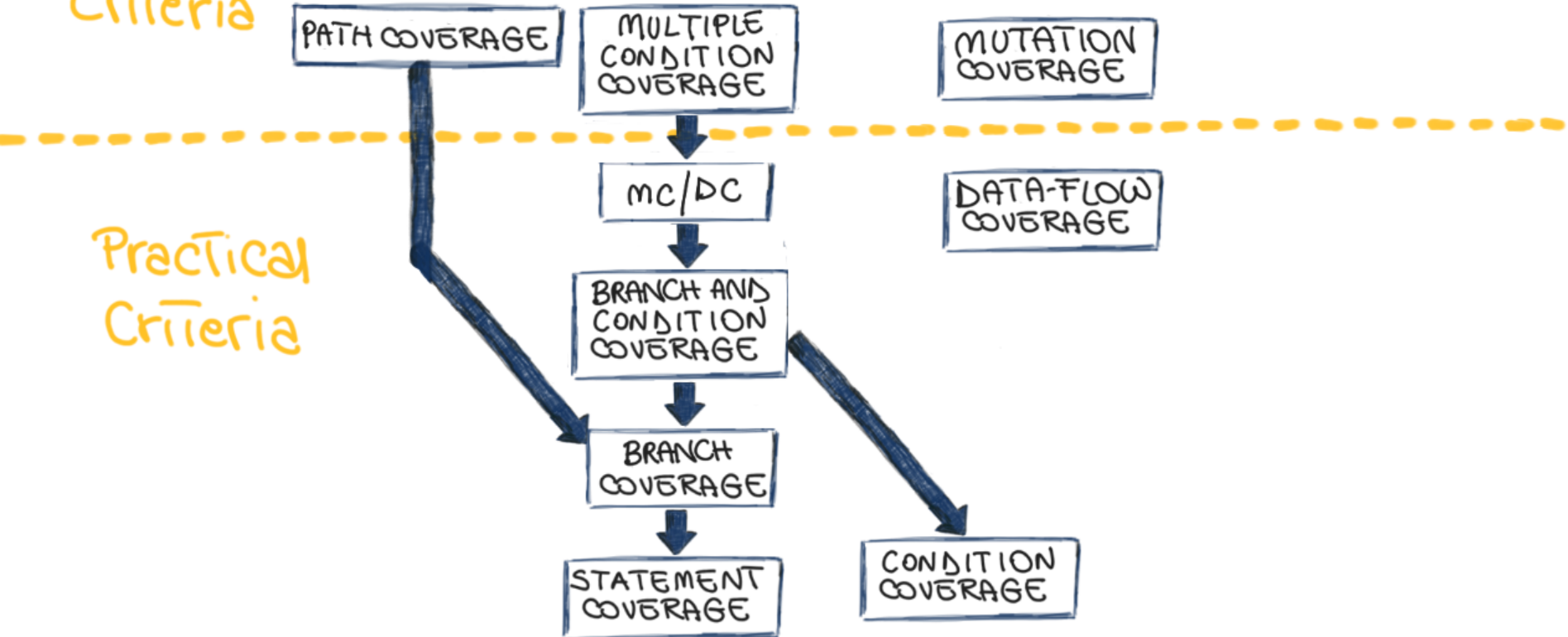
BRANCH AND  
CONDITION  
COVERAGE

BRANCH  
COVERAGE

CONDITION  
COVERAGE

STATEMENT  
COVERAGE

Practical  
Criteria







Write a faulty program  $P$  and two test suites  $T_1$  and  $T_2$  for  $P$ , such that:

1.  $T_1$  achieves 100% branch coverage but does not reveal the fault in  $P$
2.  $T_2$  achieves 100% statement coverage, does not achieve 100% branch coverage and reveals the fault in  $P$



*Write a faulty program  $P$  such that:*

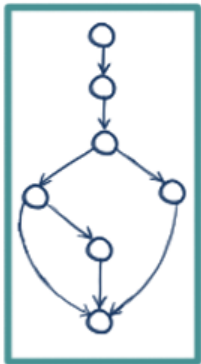
- 1. Any test suite that achieves 100% statement coverage reveals the fault in  $P$*
- 2. It is possible to write a test suite that achieves 100% branch coverage and does not reveal the fault in  $P$*



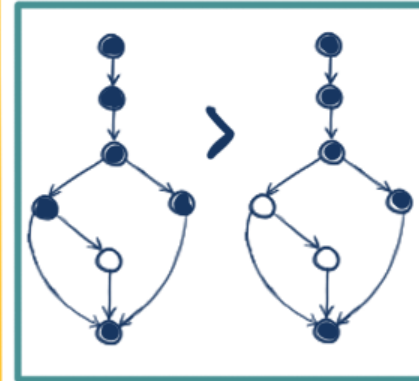
Write a faulty program  $P$  such that:

1. Any test suite that achieves 100% path coverage reveals the fault in  $P$
2. It is possible to write a test suite that achieves 100% branch coverage and does not reveal the fault in  $P$

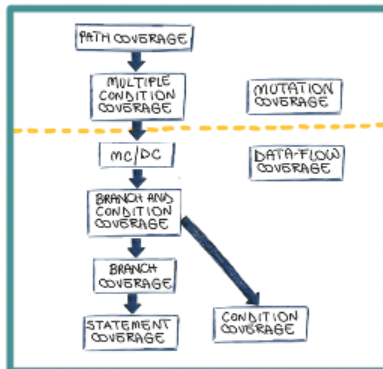
# WHITE-BOX TESTING SUMMARY



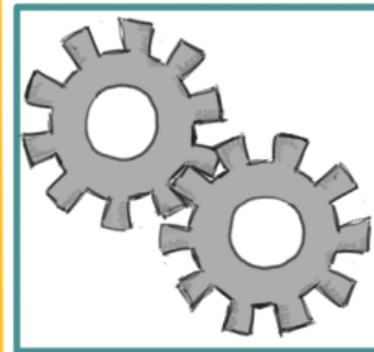
Works on a formal model



Comparable



Two broad classes:  
Practical  
Theoretical



Fully automatable