

# *Live Variables Analysis*

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# Outline

- Live Variables Analysis
- Strongly Live Variables Analysis
- Some Observations

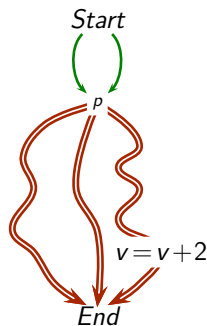
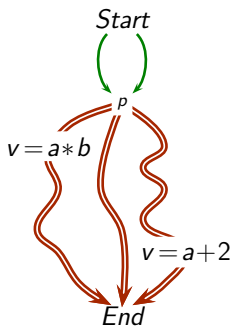
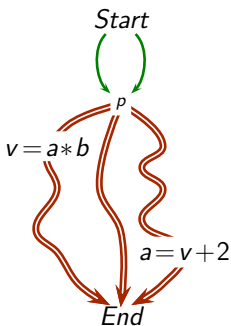


*Part 1*

# *Live Variables Analysis*

## Defining Live Variables Analysis

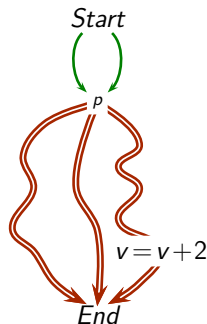
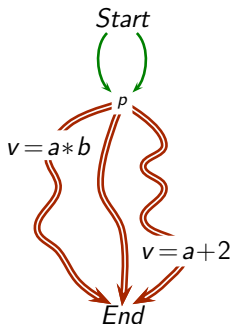
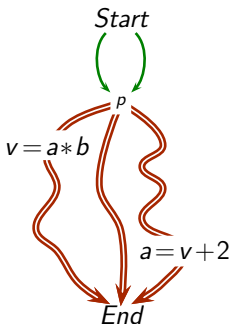
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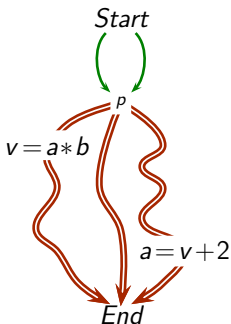
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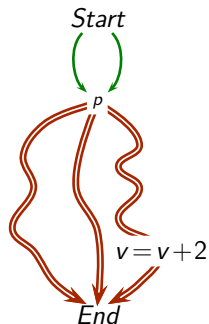
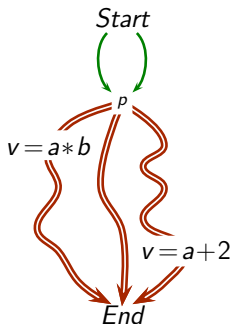
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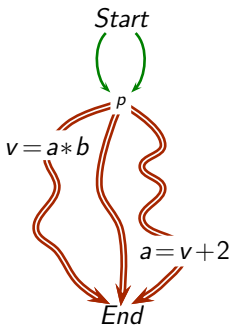
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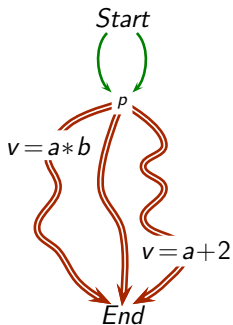
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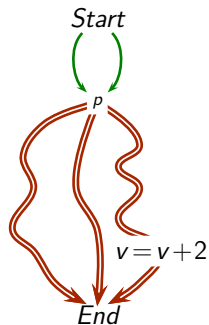
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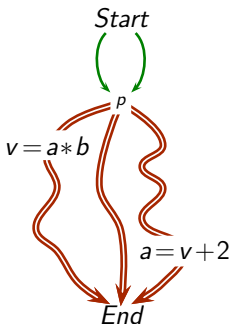


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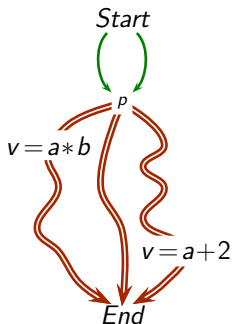
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Path based specification

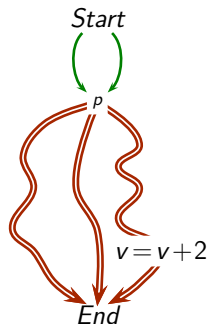
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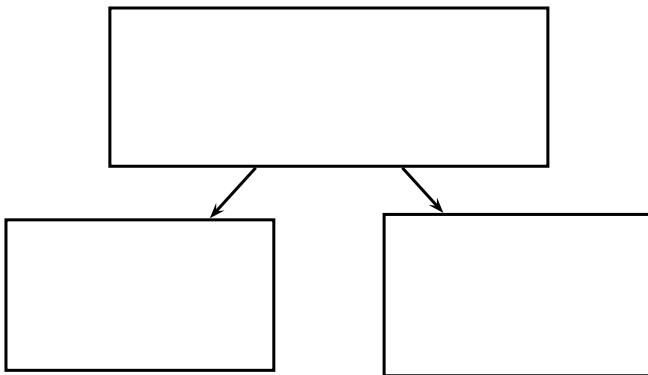


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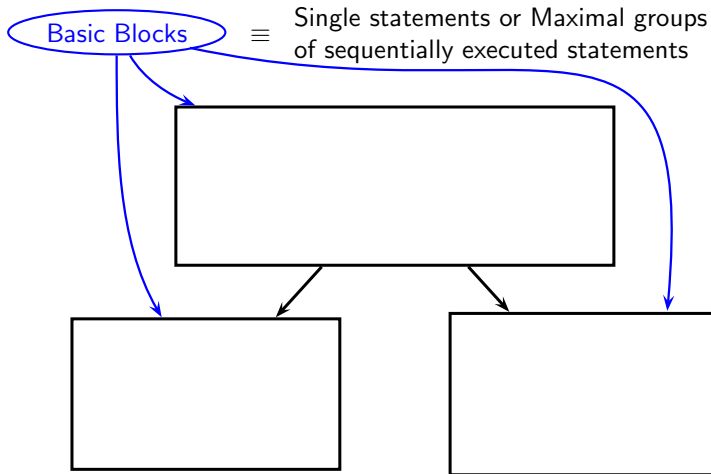




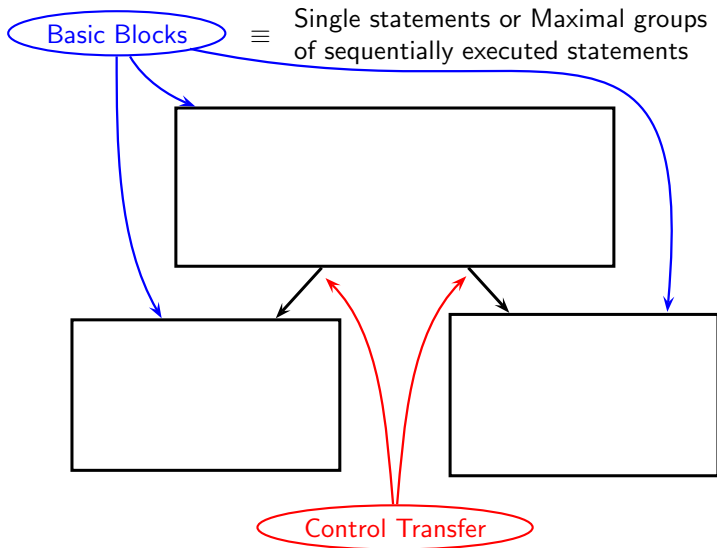
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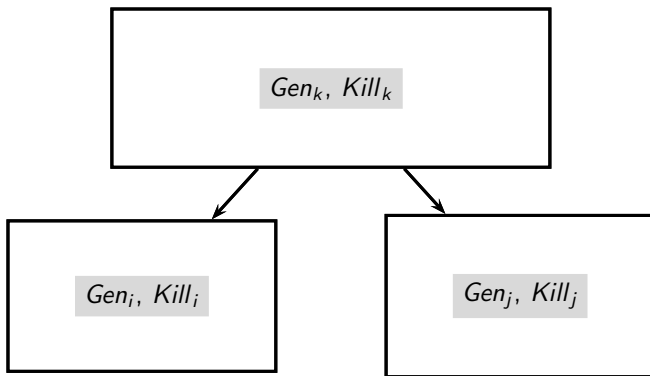
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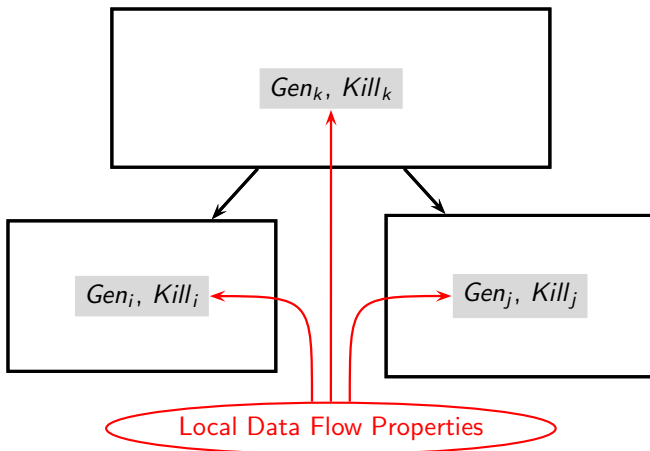
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# Local Data Flow Properties for Live Variables Analysis

$$\begin{aligned} Gen_n &= \{ v \mid \text{variable } v \text{ is used in basic block } n \text{ and} \\ &\quad \text{is not preceded by a definition of } v \} \\ Kill_n &= \{ v \mid \text{basic block } n \text{ contains a definition of } v \} \end{aligned}$$




# Local Data Flow Properties for Live Variables Analysis

r-value occurrence

Value is only read, e.g.  $x, y, z$  in

$x.sum = y.data + z.data$


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Value is modified e.g.  $y$  in

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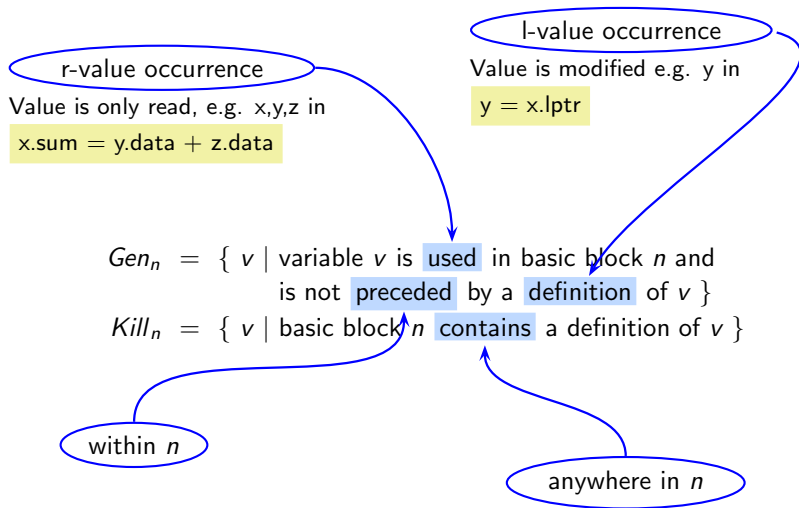
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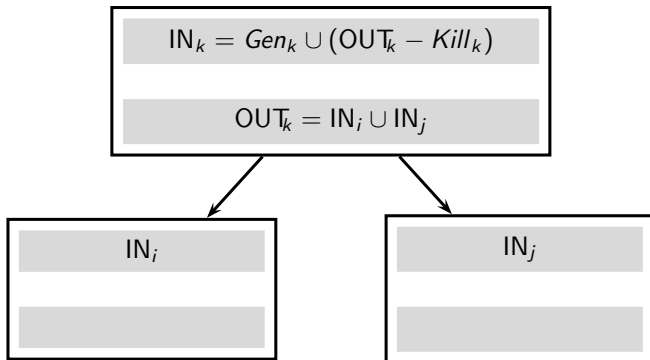
within  $n$



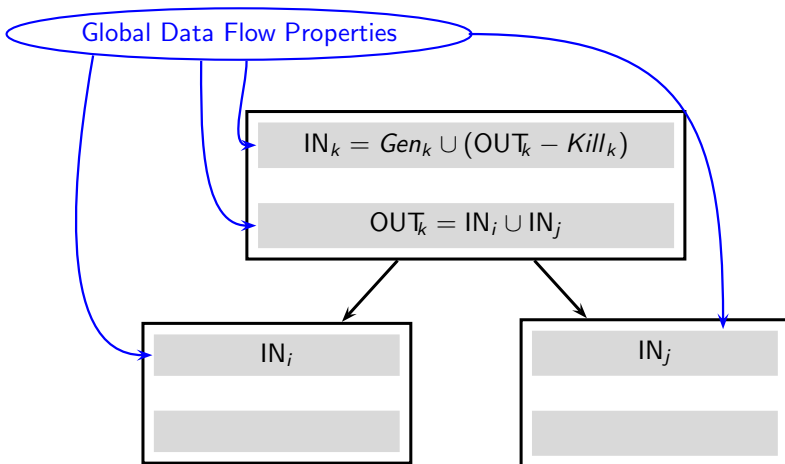
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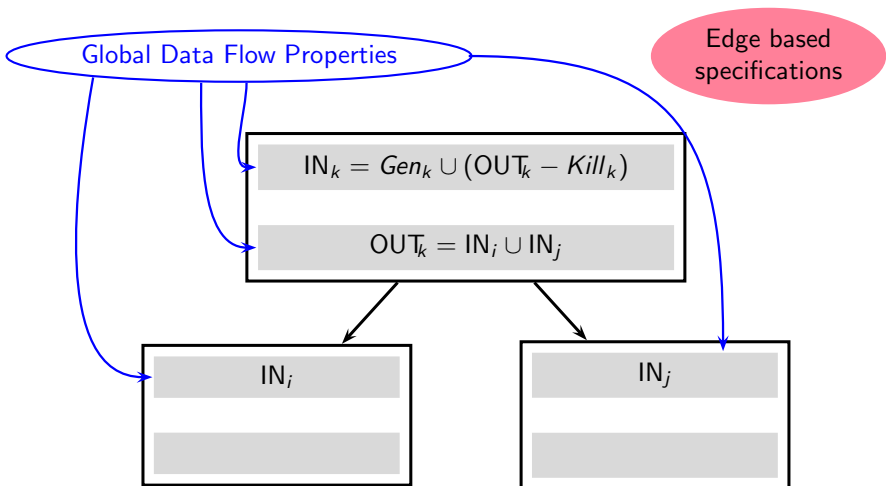
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# Data Flow Equations For Live Variables Analysis

$$\begin{aligned} \text{IN}_n &= (\text{OUT}_n - \text{Kill}_n) \cup \text{Gen}_n \\ \text{OUT}_n &= \begin{cases} \text{BI} & n \text{ is } \textit{End} \text{ block} \\ \bigcup_{s \in \textit{succ}(n)} \text{IN}_s & \text{otherwise} \end{cases} \end{aligned}$$



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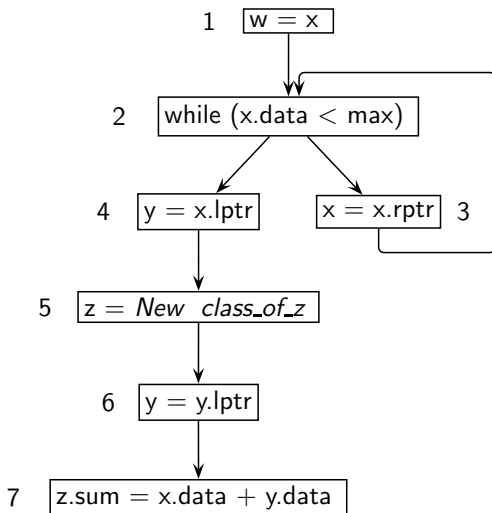
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- $\text{IN}_n$  and  $\text{OUT}_n$  are sets of variables
- $BI$  is boundary information representing the effect of calling contexts
  - ▶  $\emptyset$  for local variables except for the values being returned
  - ▶ set of global variables used further in any calling context (can be safely approximated by the set of all global variables)





## Data Flow Equations for Our Example



$$IN_1 = (OUT_1 - Kill_1) \cup Gen_1$$

$$OUT_1 = IN_2$$

$$IN_2 = (OUT_2 - Kill_2) \cup Gen_2$$

$$OUT_2 = IN_3 \cup IN_4$$

$$IN_3 = (OUT_3 - Kill_3) \cup Gen_3$$

$$OUT_3 = IN_2$$

$$IN_4 = (OUT_4 - Kill_4) \cup Gen_4$$

$$OUT_4 = IN_5$$

$$IN_5 = (OUT_5 - Kill_5) \cup Gen_5$$

$$OUT_5 = IN_6$$

$$IN_6 = (OUT_6 - Kill_6) \cup Gen_6$$

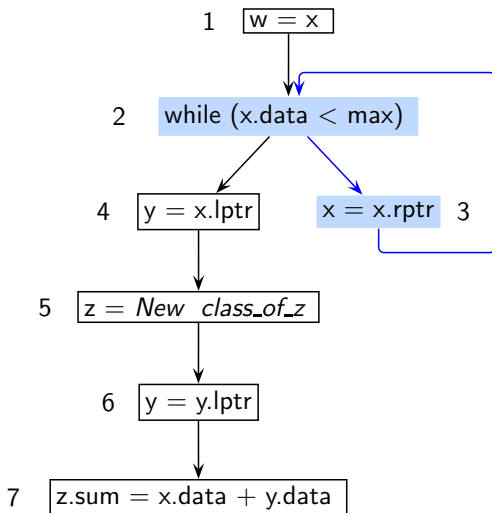
$$OUT_6 = IN_7$$

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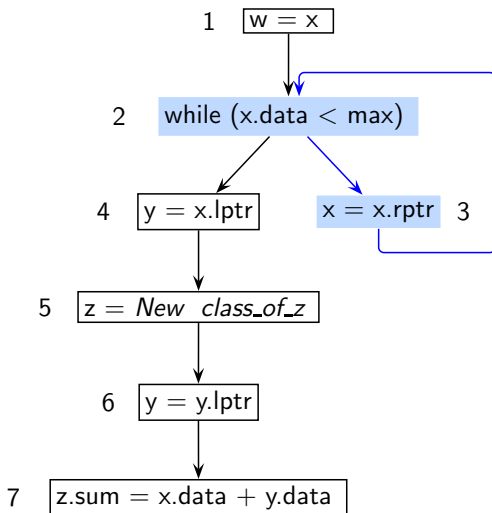
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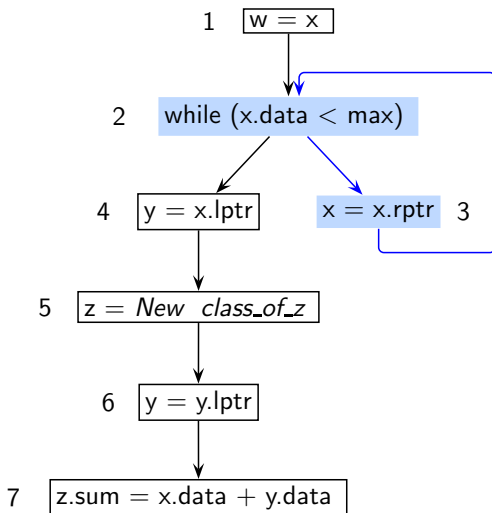
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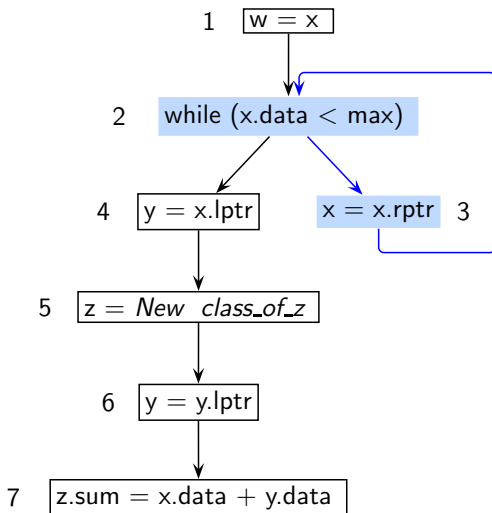
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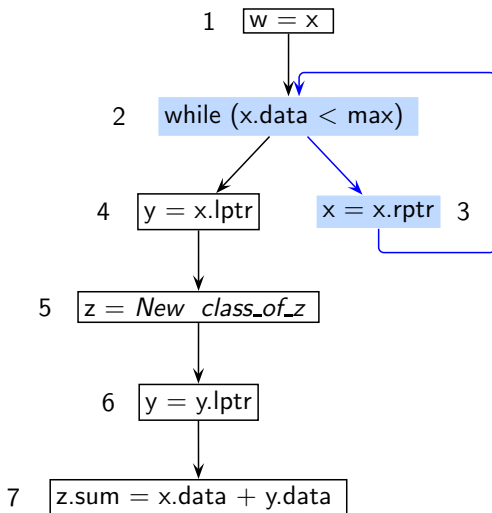
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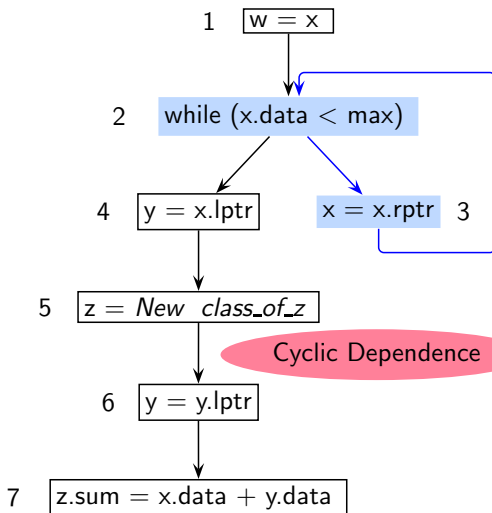
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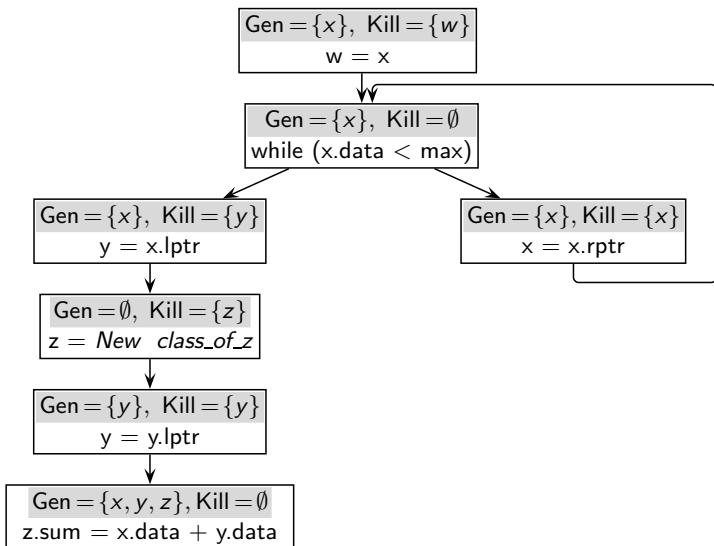
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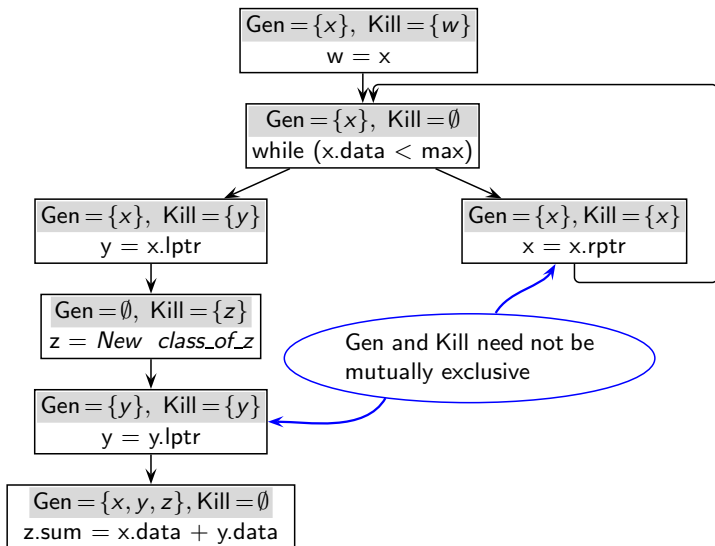


## Performing Live Variables Analysis

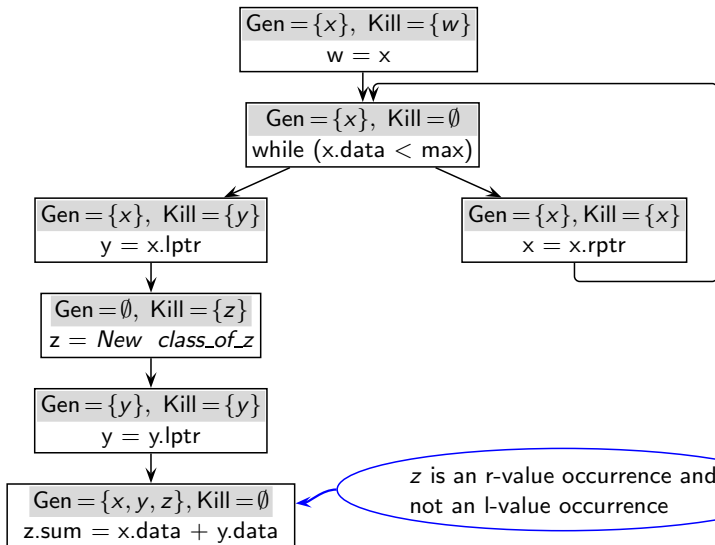




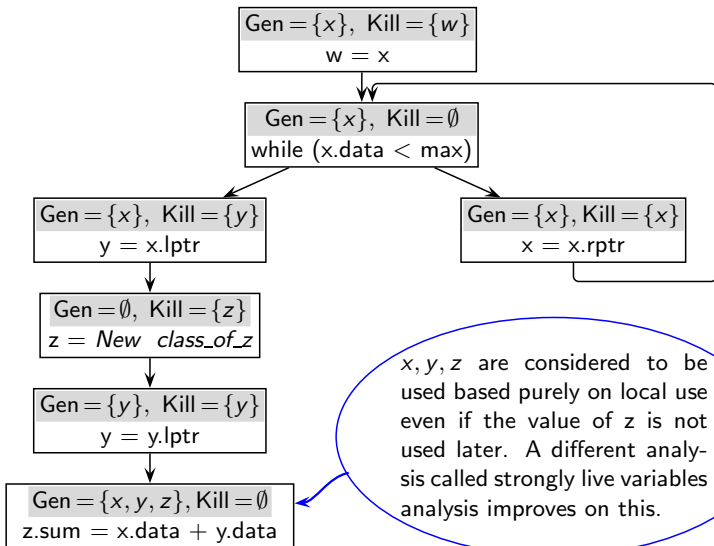
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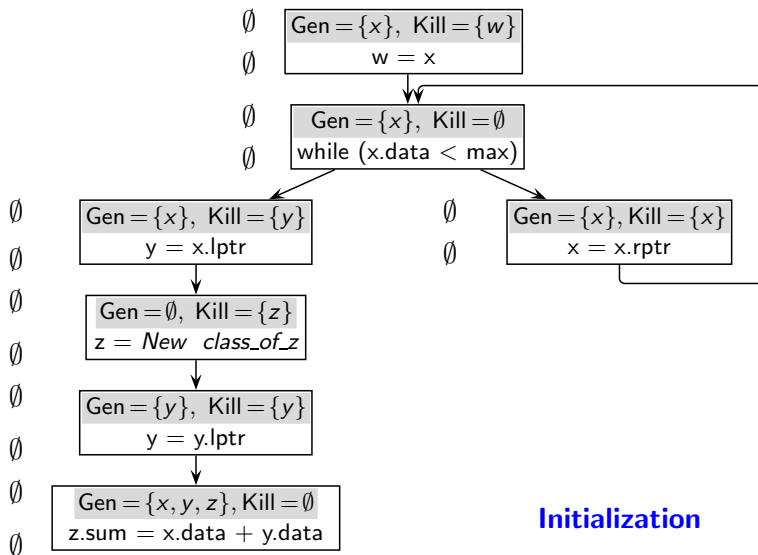
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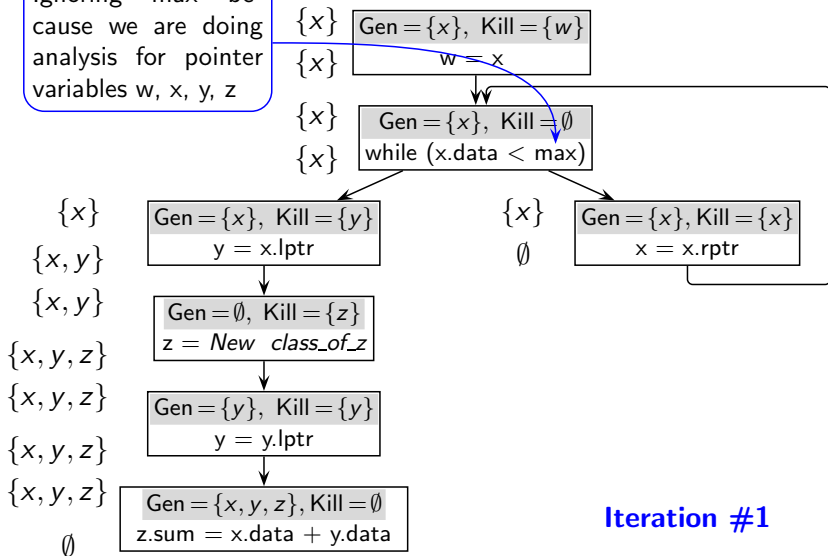


Initialization



# Performing Live Variables Analysis

Ignoring max because we are doing analysis for pointer variables w, x, y, z



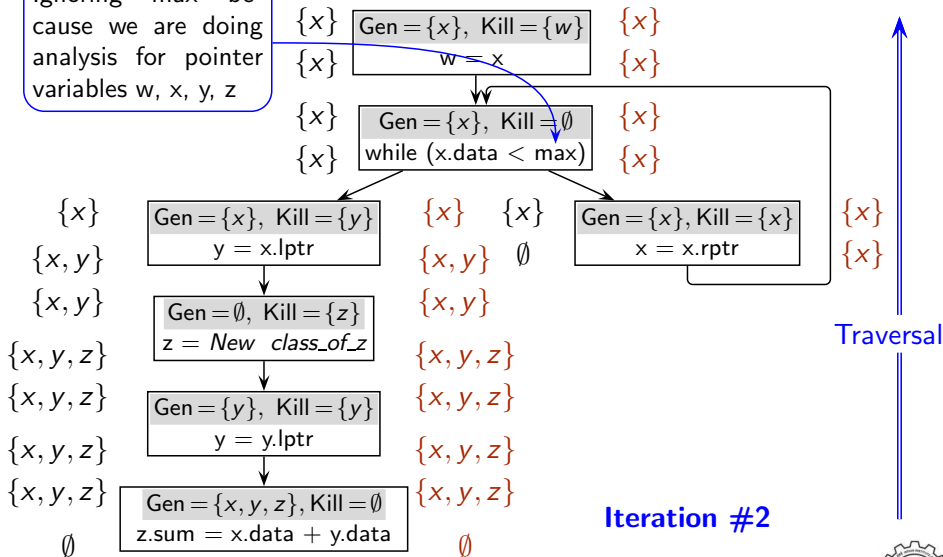
Traversal

Iteration #1



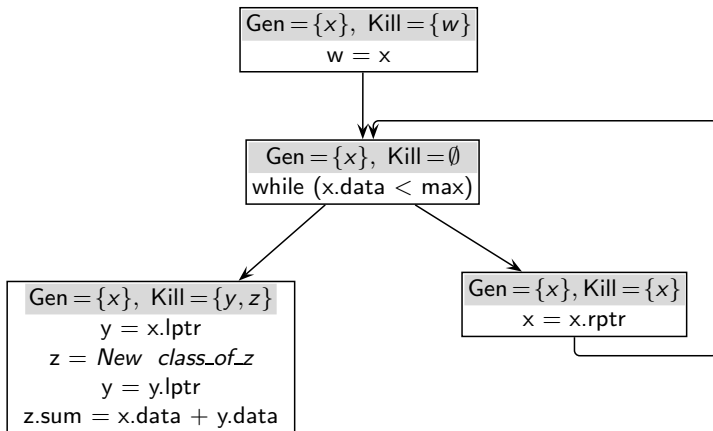
# Performing Live Variables Analysis

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## Performing Live Variables Analysis

Local data flow properties when basic blocks contain multiple statements



## Local Data Flow Properties for Live Variables Analysis

$$IN_n = Gen_n \cup (OUT_n - Kill_n)$$

- $Gen_n$  : Use not preceded by definition
- $Kill_n$  : Definition anywhere in a block





# Local Data Flow Properties for Live Variables Analysis

$$IN_n = Gen_n \cup (OUT_n - Kill_n)$$

- $Gen_n$  : Use not preceded by definition

Upwards exposed use

- $Kill_n$  : Definition anywhere in a block

Stop the effect from being propagated across a block



## Using Data Flow Information of Live Variables Analysis

- Used for register allocation

If variable  $x$  is live in a basic block  $b$ , it is a potential candidate for register allocation



# Using Data Flow Information of Live Variables Analysis

- Used for register allocation

If variable  $x$  is live in a basic block  $b$ , it is a potential candidate for register allocation

- Used for dead code elimination

If variable  $x$  is not live after an assignment  $x = \dots$ , then the assignment is redundant and can be deleted as dead code



*Part 2*

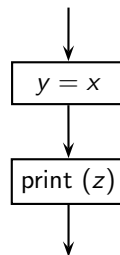
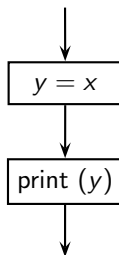
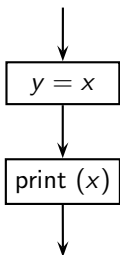
# *Strongly Live Variables Analysis*

## Strongly Live Variables Analysis

- A variable is strongly live if
  - ▶ it is used in a statement other than assignment statement, or (same as simple liveness)
  - ▶ it is used in an assignment statement defining a variable that is strongly live (different from simple liveness)
- Killing: An assignment statement, an input statement, or BI (this is same as killing in simple liveness)
- Generation: A direct use or a use for defining values that are strongly live (this is different from generation in simple liveness)

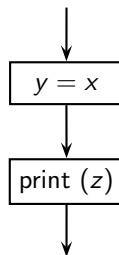
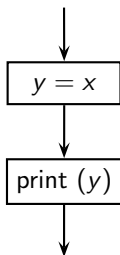
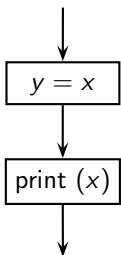


## Understanding Strong Liveness



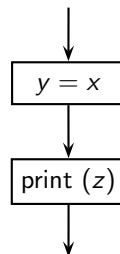
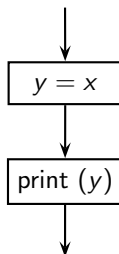
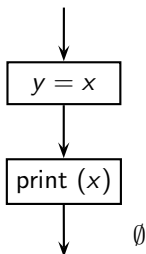
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Strong  
Liveness



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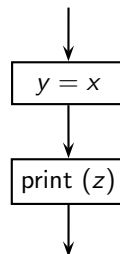
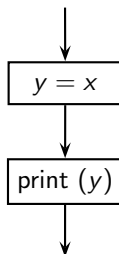
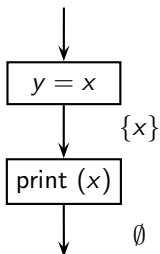
Strong  
Liveness



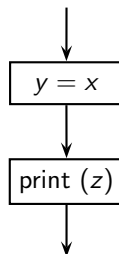
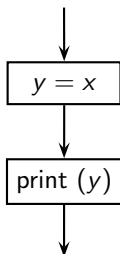
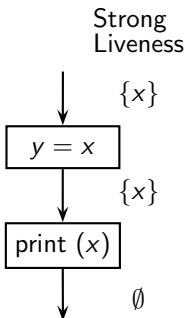


# Understanding Strong Liveness

Strong  
Liveness



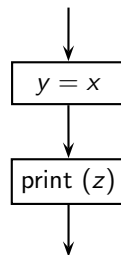
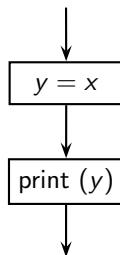
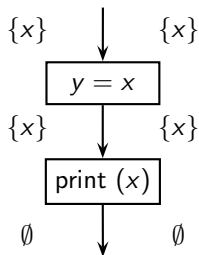
# Understanding Strong Liveness



# Understanding Strong Liveness

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Liveness

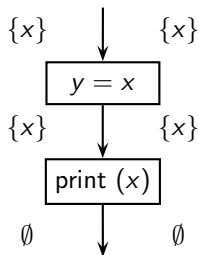
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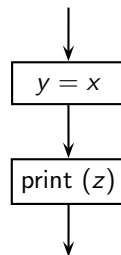
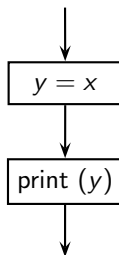
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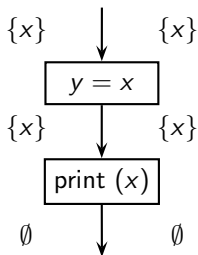
Same



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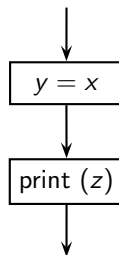
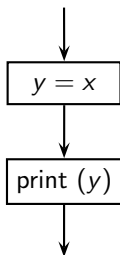
Simple  
Liveness

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Same

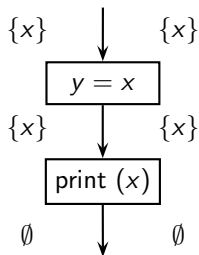
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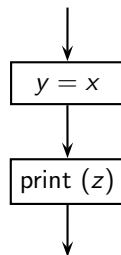
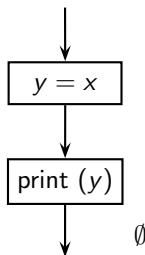
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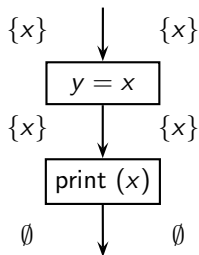
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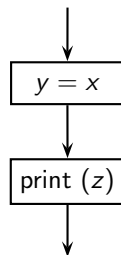
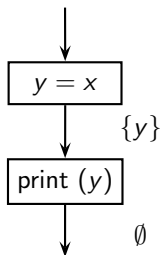
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Liveness

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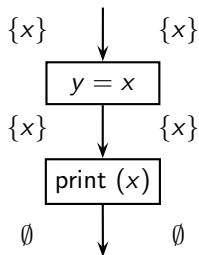
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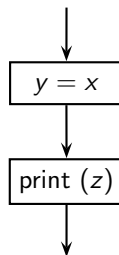
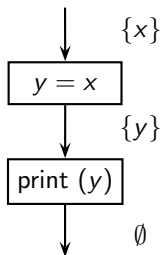
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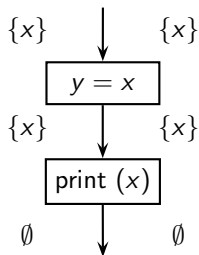




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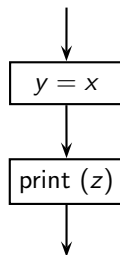
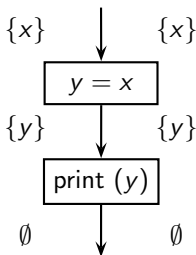
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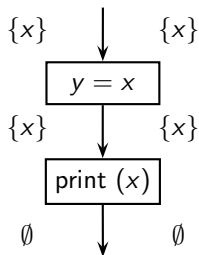
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## Understanding Strong Liveness

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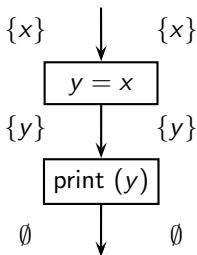
Strong  
Liveness



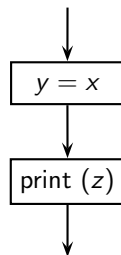
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Liveness

Strong  
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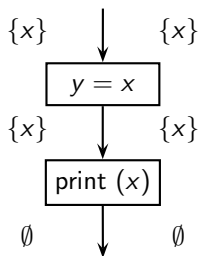


Same



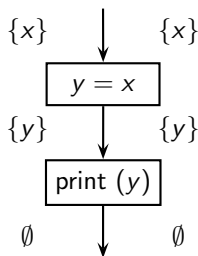
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Simple Liveness      Strong Liveness



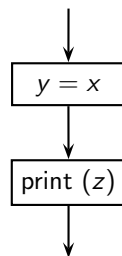
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Simple Liveness      Strong Liveness



Same

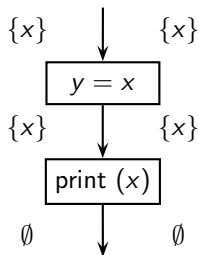
Strong Liveness



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Simple  
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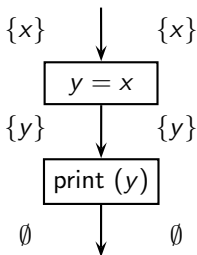
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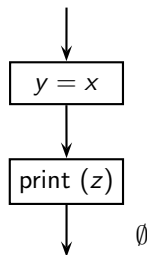
Simple  
Liveness

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Same

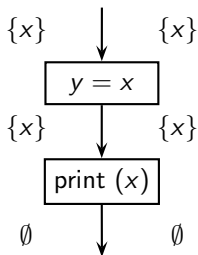
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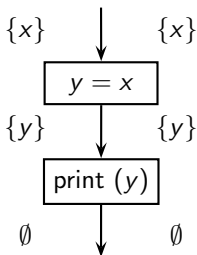
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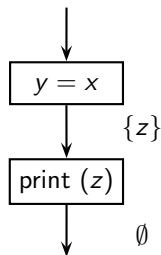
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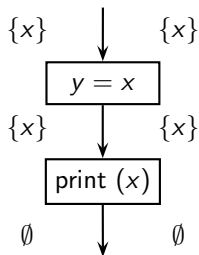
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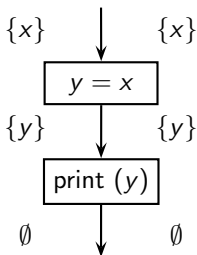
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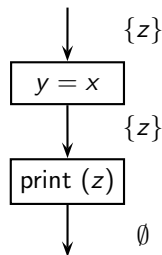
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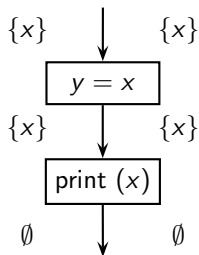
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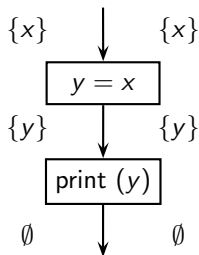
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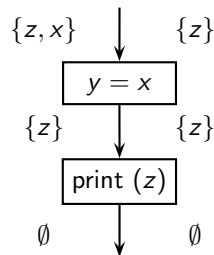
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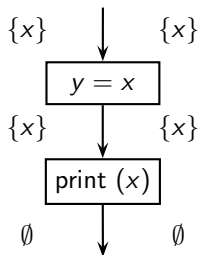
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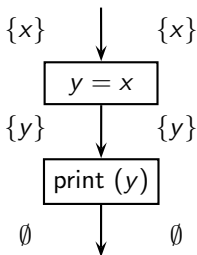
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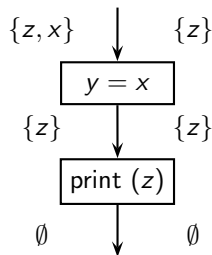
Same

Simple Liveness      Strong Liveness



Same

Simple Liveness      Strong Liveness



Different





# Live Variables Analysis: Simple and Strong Liveness

- A variable is live at a program point if its current value is likely to be used later



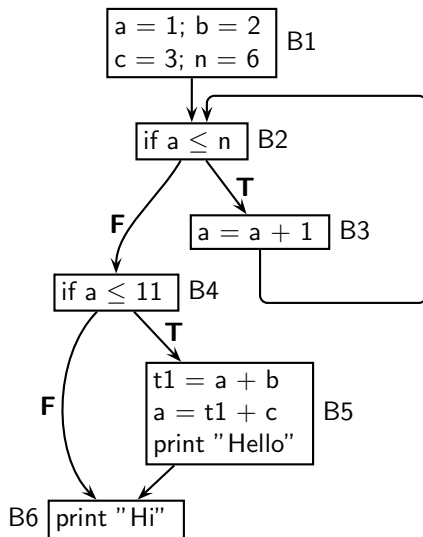
# Live Variables Analysis: Simple and Strong Liveness

- A variable is live at a program point if its current value is likely to be used later
- We want to compute the smallest set of variables that are live



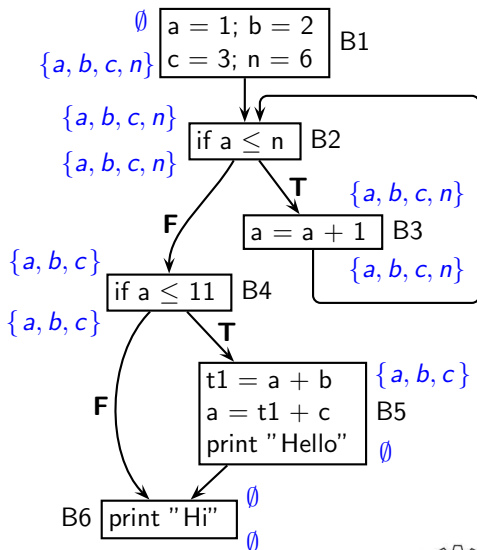
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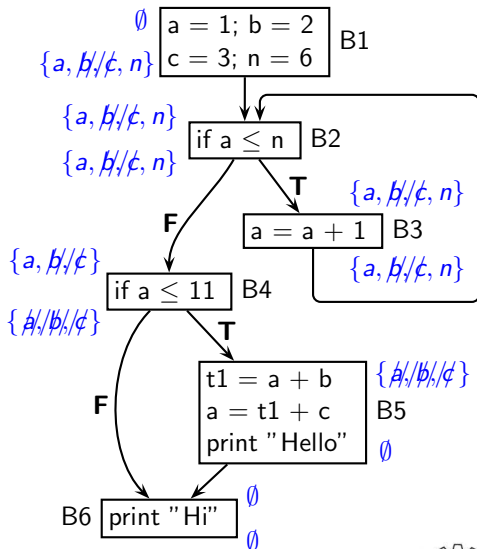
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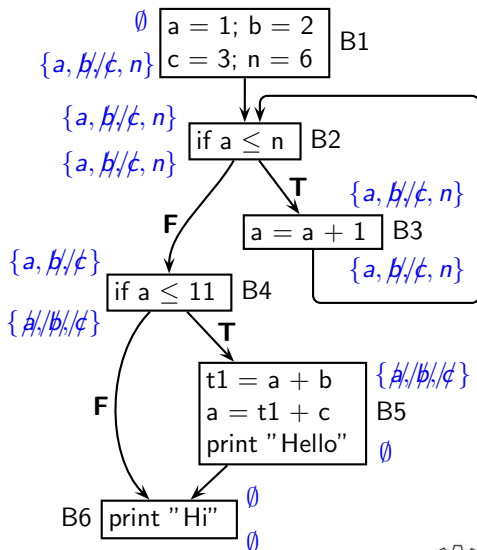
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- We want to compute the smallest set of variables that are live
- Simple liveness considers every use of a variable as useful
- Strong liveness checks the liveness of the result before declaring the operands to be live
- Strong liveness is more precise than simple liveness



# Data Flow Equations for Strongly Live Variables Analysis

$$IN_n = f_n(OUT_n)$$

$$OUT_n = \begin{cases} BI & n \text{ is } End \\ \bigcup_{s \in succ(n)} IN_s & \text{otherwise} \end{cases}$$

where,

$$f_n(X) = \begin{cases} (X - \{y\}) \cup (Opd(e) \cap \mathbb{V}ar) & n \text{ is } y = e, e \in \mathbb{E}xpr, y \in X \\ X - \{y\} & n \text{ is } input(y) \\ X \cup \{y\} & n \text{ is } use(y) \\ X & \text{otherwise} \end{cases}$$



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If  $y$  is not strongly live, the assignment is skipped using the “otherwise” clause





## *Part 3*

# *Some Observations*

# What Does Data Flow Analysis Involve?

- Defining the analysis.
- Formulating the analysis.
- Performing the analysis.



# What Does Data Flow Analysis Involve?

- Defining the analysis. Define the properties of execution paths
- Formulating the analysis.
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# What Does Data Flow Analysis Involve?

- **Defining the analysis.** Define the properties of execution paths
- **Formulating the analysis.** Define data flow equations
  - ▶ Linear simultaneous equations on sets rather than numbers
  - ▶ Later we will generalize the domain of values
- **Performing the analysis.**



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- **Formulating the analysis.** Define data flow equations
  - ▶ Linear simultaneous equations on sets rather than numbers
  - ▶ Later we will generalize the domain of values
- **Performing the analysis.** Solve data flow equations for the given program flow graph
- Many unanswered questions  
Initial value? Termination? Complexity? Properties of Solutions?



# Iterative Solution of Linear Simultaneous Equations

- Simultaneous equations represented in the form of the product of a matrix of coefficients (**A**) with the vector of unknowns (**x**)

$$\mathbf{Ax} = \mathbf{b}$$

- Start with approximate values
- Compute new values repeatedly from old values
- Two classical methods
  - ▶ Gauss-Seidel Method (Gauss: 1823, 1826), (Seidel: 1874)
  - ▶ Jacobi Method (Jacobi: 1845)



## Our Method of Performing Data Flow Analysis

- Round robin iteration
- Essentially Jacobi method
- Unknowns are the data flow variables  $IN_i$  and  $OUT_i$
- Domain of values is not numbers
- Computation in a fixed order
  - ▶ either forward (reverse post order) traversal, or
  - ▶ backward (post order) traversal

over the control flow graph





# Soundness and Precision of Live Variables Analysis

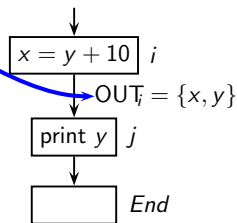
Consider dead code elimination based on liveness information



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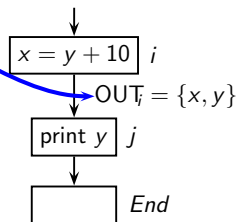
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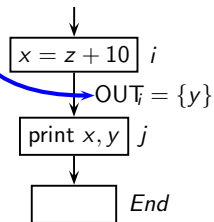
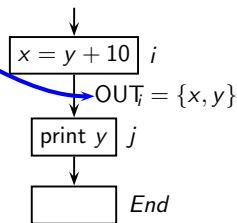
- Spurious inclusion of a non-live variable
  - ▶ A dead assignment may not be eliminated
  - ▶ Solution is sound but may be imprecise



## Soundness and Precision of Live Variables Analysis

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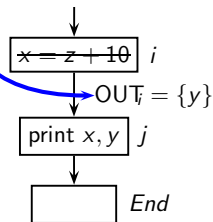
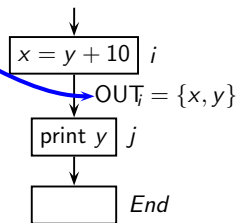
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- Spurious exclusion of a live variable
  - ▶ A useful assignment may be eliminated
  - ▶ Solution is unsound



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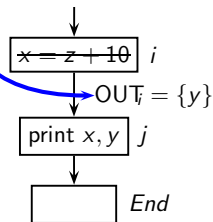
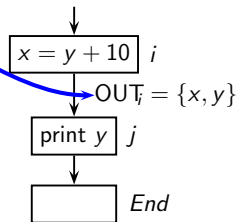
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- Given  $L_2 \supseteq L_1$  representing liveness information

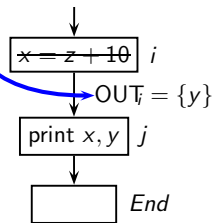
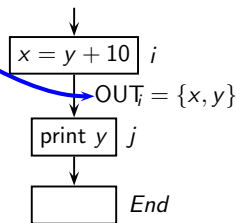
- ▶ Using  $L_2$  in place of  $L_1$  is sound
- ▶ Using  $L_1$  in place of  $L_2$  may not be sound



# Soundness and Precision of Live Variables Analysis

Consider dead code elimination based on liveness information

- Spurious inclusion of a non-live variable
  - ▶ A dead assignment may not be eliminated
  - ▶ Solution is sound but may be imprecise
- Spurious exclusion of a live variable
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  - ▶ Solution is unsound
- Given  $L_2 \supseteq L_1$  representing liveness information
  - ▶ Using  $L_2$  in place of  $L_1$  is sound
  - ▶ Using  $L_1$  in place of  $L_2$  may not be sound
- The smallest set of all live variables is most precise
  - ▶ Since liveness sets grow (confluence is  $\cup$ ), we choose  $\emptyset$  as the initial conservative value



# Termination, Convergence, and Complexity

- For live variables analysis,
    - ▶ The set of all variables is finite, and
    - ▶ the confluence operation (i.e. meet) is union, hence
    - ▶ the set associated with a data flow variable can only grow
- ⇒ Termination is guaranteed





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- $\Rightarrow$  Termination is guaranteed
- Since initial value is  $\emptyset$ , live variables analysis converges on the smallest set



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- How many iterations do we need for reaching the convergence?

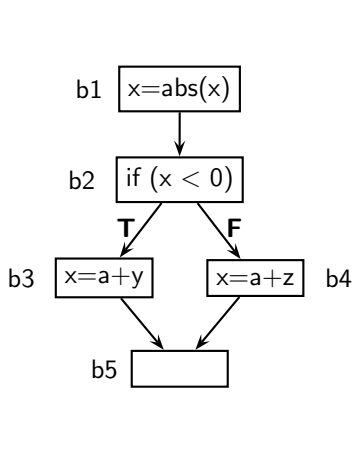


# Termination, Convergence, and Complexity

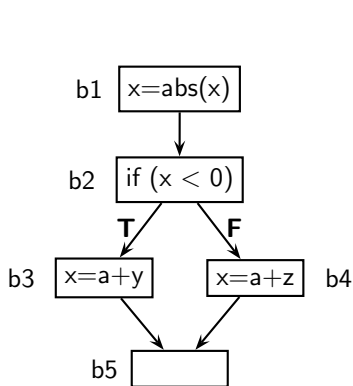
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- ⇒ Termination is guaranteed
- Since initial value is  $\emptyset$ , live variables analysis converges on the smallest set
- How many iterations do we need for reaching the convergence?
- Going beyond live variables analysis
  - ▶ Do the sets always grow for other data flow frameworks?
  - ▶ What is the complexity of round robin analysis for other analyses?



# Conservative Nature of Analysis (1)



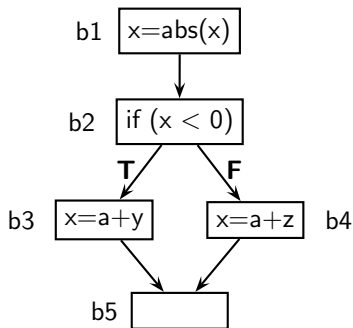
## Conservative Nature of Analysis (1)



- `abs(n)` returns the absolute value of `n`



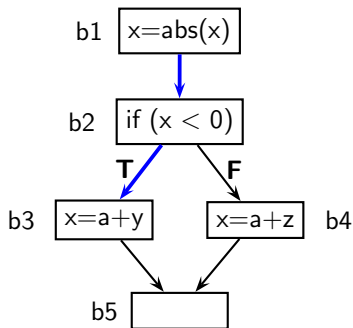
## Conservative Nature of Analysis (1)



- `abs(n)` returns the absolute value of `n`
- Is `y` live on entry to block `b2`?



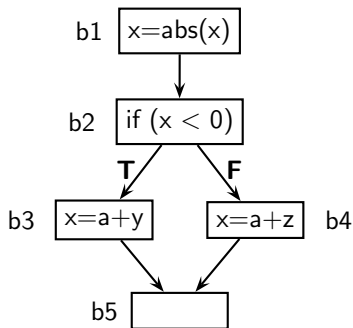
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Path  $b1 \rightarrow b2 \rightarrow b3$  is an infeasible execution path



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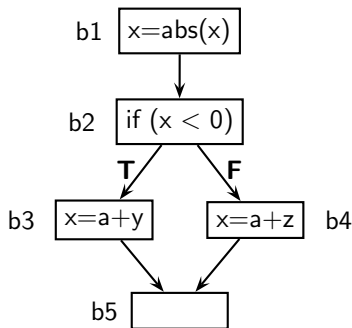
*All branch outcomes are possible*

$\Rightarrow$  Consider every path in CFG as a potential execution path





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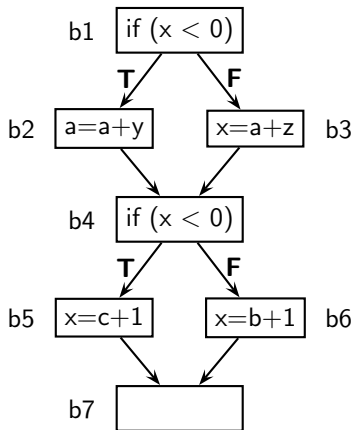
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- Our analysis concludes that  $y$  is live on entry to block  $b2$

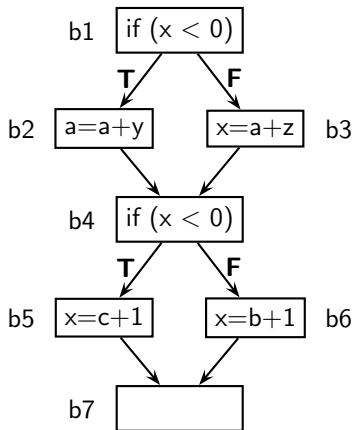


## Conservative Nature of Analysis (2)

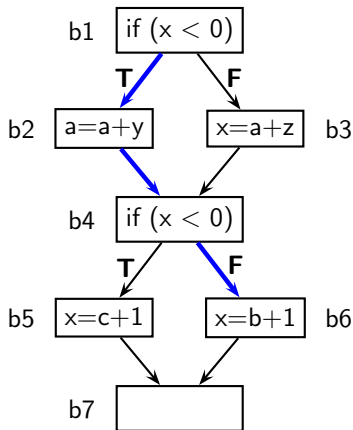


## Conservative Nature of Analysis (2)

- Is b live on entry to block b2?



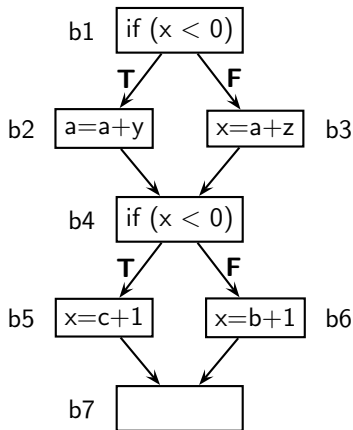
## Conservative Nature of Analysis (2)



- Is b live on entry to block b2?
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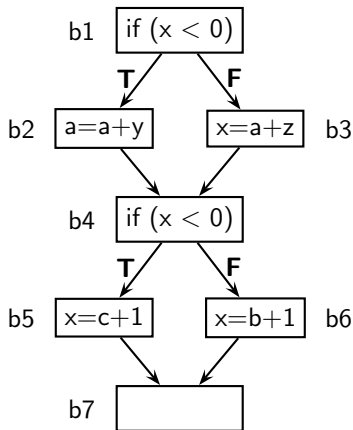
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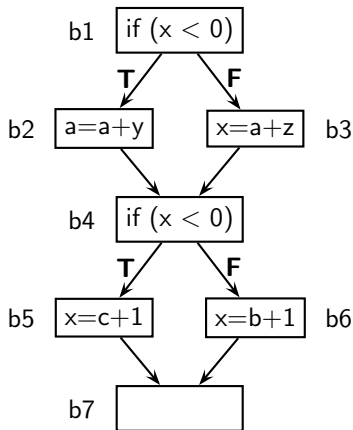
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 $\Rightarrow$  our analysis is *path insensitive*  
Note: It is *flow sensitive* (i.e. information is computed for every control flow points)



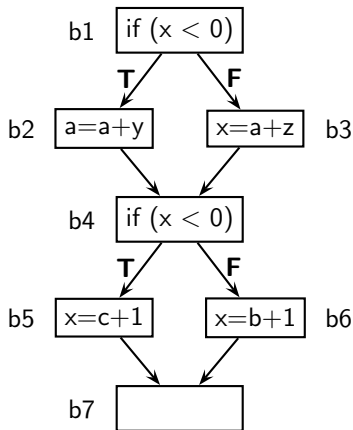
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- Our analysis concludes that b is live at the entry of b2
- Is c live at the entry of b3?





## Conservative Nature of Analysis at Intraprocedural Level

- We assume that all paths are potentially executable
- Our analysis is path insensitive
  - ▶ The data flow information at a program point  $p$  is path insensitive
    - information at  $p$  is merged along all paths reaching  $p$
  - ▶ The data flow information reaching  $p$  is computed path insensitively
    - information is merged at all shared points in paths reaching  $p$
    - may generate spurious information due to non-distributive flow functions



# Conservative Nature of Analysis at Interprocedural Level

- Context insensitivity
  - ▶ Merges of information across all calling contexts
- Flow insensitivity
  - ▶ Disregards the control flow

More about it later



## Looking Ahead

The exciting word of pointer analysis beckons us

